# NEW STABILITY AND STABILIZATION CRITERIA FOR CONTINUOUS SYSTEM WITH TIME DELAY

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Communicated by Dafik

MSC 2020 Classifications: 49K15

Keywords and phrases: Stability, Linear matrix inequalities.

**Abstract** This paper address the control problem for a class of continuous time system. In particular, by employing a novel Lyapunov functional together with Jenson's lemma, control law is designed in terms of the solution of certain linear matrix inequalities which makes the considered system asymptotically stable which can be easily solved by using the available software. Finally, a numerical example is given to demonstrate the effectiveness and applicability of the proposed design approach.

# **1** Introduction

Linear problems play an very important role in science and technology as many practical systems are dynamical in nature [6]. Also, time delay is one of the common phenomenon often encountered in real dynamical systems such as cascaded electrical networks, robotics systems, chemical processes, turbojet engines, traffic networks, telecommunication systems due to measurement and so on [7, 5]. Stability criteria may be of two types, namely delay independent and delay-independent [1, 2]. They may be time invariant or delay dependent system and is suitable for both linear and non linear systems. The stability analysis of the system varies from time to time. Stability analysis is the key characteristic that will decide the nature of the control systems and it varies from time to time [3, 4]. This is obtained by using Jenson's inequality and Newton Leibniz rule [8, 10]. The concept of Lyapunov function have been studied intensively and exclusively by Mathematicians and scientists to investigate the stability of any dynamical system [9]. In recent scenario, the analysis of stability of linear and non linear systems is very essential.

In this paper, a set of sufficient condition is obtained for stability performance of the system by using the new Lyapunov-Krasovskii functional and LMI technique which are dependent on the lower and upper bounds of the time-varying delay. Then, we design a state feedback control law based on the obtained LMI conditions.

**Notations:** Throughout the article, the superscripts T and (-1) stand for matrix transposition and matrix inverse, respectively  $R^n$  denotes the n-dimensional Euclidean space. $R^{n \times n}$  denote the set of all  $n \times n$  real matrices. P > 0 (respectively, P < 0) means that P is positive definite (respectively, negative definite). I and 0 represent identity matrix and zero matrix with compatible dimensions.

## 2 Problem Formulation and Preliminaries

In many real-world problems, time delays are frequently encountered in its dynamics and many researchers have studied problems of designing control law for the dynamical systems with input delays. In this paper, we consider the linear continuous system with time varying delay in the following form

$$\dot{x}(t) = Ax(t) + A_d x(t - d(t)) + Bu(t),$$

$$x(t) = \phi(t), \quad \forall t \in [-d_2, 0],$$
(2.1)

where  $x(t) \in \mathbb{R}^n$ ,  $u(t) \in \mathbb{R}^m$ , represents the state vector, the control input vector respectively; A,A<sub>d</sub> and B are known real matrices of appropriate dimensions. Further, the unknown timevarying delay d(t) satisfies the following conditions

$$0 \le d_1 \le d(t) \le d_2, \quad \dot{d}(t) \le \mu < 1, \tag{2.2}$$

where  $d_1$ ,  $d_2$  and  $\mu$  are positive constants and  $\phi(t)$  is the initial function  $\forall t \in [-d_2, 0]$ . The control input u(t) can be described as

$$u(t) = Kx(t). \tag{2.3}$$

**Definition 2.1.** [1] Let x = 0 be an equilibrium point of  $\dot{x} = f(x)$ ,  $f : D \to R^n$ . Let  $V : D \to R^+$  be a continuously differentiable function such that,

- (i) V(0) = 0 and V(x) > 0 in  $D \{0\}$ .
- (ii)  $\dot{V}(x) \le 0$  in  $D \{0\}$ . Then x = 0 is stable.
- (iii)  $\dot{V}(x) < 0$  in  $D \{0\}$ . Then x = 0 is asymptotically stable.

**Lemma 2.2.** [7] Given a positive definite matrix  $Z \in \mathbb{R}^{n \times n}$ ,  $Z = Z^T > 0$  and scalars  $\tau_1 < \tau(t) < \tau_2$ , for vector function  $x(t) = [x_1(t) x_2(t) \dots x_n(t)]^T$ , we have

$$\begin{split} &-\int_{t-\tau_{2}}^{t-\tau_{1}} \dot{x}^{T}(s) Z \dot{x}(s) ds &\leq -\frac{1}{\tau_{2}-\tau_{1}} \left( \int_{t-\tau_{2}}^{t-\tau_{1}} \dot{x}(s) ds \right)^{T} Z \left( \int_{t-\tau_{2}}^{t-\tau_{1}} \dot{x}(s) ds \right), \\ &-\int_{-\tau_{2}}^{-\tau_{1}} \int_{t+\theta}^{t} \dot{x}^{T}(s) Z \dot{x}(s) ds d\theta &\leq -\frac{2}{\tau_{2}^{2}-\tau_{1}^{2}} \left( \int_{t-\tau_{2}}^{t-\tau_{1}} \int_{t+\theta}^{t} \dot{x}(s) ds d\theta \right)^{T} Z \\ &\qquad \left( \int_{t-\tau_{2}}^{t-\tau_{1}} \int_{t+\theta}^{t} \dot{x}(s) ds d\theta \right). \end{split}$$

**Lemma 2.3.** [8] Given the constant matrices  $\Sigma_1$ ,  $\Sigma_2$ ,  $\Sigma_3$ , where  $\Sigma_1 = \Sigma_1^T$  and  $0 < \Sigma_2 = \Sigma_2^T$ . Then  $\Sigma_1 + \Sigma_3^T \Sigma_2^{-1} \Sigma_3 < 0$  if and only if  $\begin{bmatrix} \Sigma_1 & \Sigma_3^T \\ \Sigma_3 & -\Sigma_2 \end{bmatrix} < 0$  or equivalently  $\begin{bmatrix} -\Sigma_2 & \Sigma_3 \\ \Sigma_3^T & \Sigma_1 \end{bmatrix} < 0$ .

#### 3 Stability and Stabilization of Time-varying Delay System

In this section, we propose a novel stability theorem for the systems with time-varying delay subsequently the result is extended to obtain the desired controller.

**Theorem 3.1.** Let the scalar  $\mu > 0$  be given. The system (2.1) with u(t) = 0 is asymptotically stable if there exists matrices P > 0  $Q_i > 0$ ,  $R_j > 0$  (*i*=1,2,3, *r*=1,2),  $N_{a,b}$ , a = b = 1, 2, 3, 4 with appropriate dimensions and delay d(t) satisfying (2.2) such that the following inequality holds;

$$[\Pi]_{9\times9} < 0, \tag{3.1}$$

where

$$\begin{split} \Pi_{1,1} &= 2PA + Q_1 + Q_2 + Q_3 + d_1 A^T R_1 A + (d_2 - d_1) A^T R_2 A + d_1 A_d^T R_1 A + 2N_{11}^T \\ &+ A^T \frac{d_1^2}{2} R_3 A + \frac{d_2^2 - d_1^2}{2} A^T R_4 A, \\ \Pi_{1,2} &= N_{11}^T + N_{12}^T + N_{12}, \\ \Pi_{1,3} &= P^T A_d^T \\ &+ d_1 A_d^T R_1 A + (d_2 - d_1) A^T R_2 A_d + N_{13} + N_{12}^T + N_{1,2}^T + \frac{d_1}{2} A^T R_3 A_d \\ &+ \frac{d_2^2 - d_1^2}{2} A^T R_4 A_d, \\ \Pi_{1,4} &= N_{13}^T + N_{1,4}^T, \\ \Pi_{1,5} &= -N_{1,4}, \\ \Pi_{1,6} &= -N_{31}, \\ \Pi_{1,7} &= -N_{21}, \\ \Pi_{2,2} &= -Q_1 - N_{1,2} - N_{22}, \\ \Pi_{2,3} &= -N_{22}^T + N_{23}^T - N_{13} + N_{23}, \\ \Pi_{2,4} &= N_{2,3}^T - N_{14} + N_{24}, \\ \Pi_{2,5} &= -N_{21}^T, \\ \Pi_{2,6} &= -N_{23}^T, \\ \Pi_{3,3} &= -(1 - \mu)Q_3 + d_1 A_d^T R_1 A_d + (d_2 - d_2) A_d^T R_2 A_d + N_{23} + N_{33} + \frac{d_1^2}{2} A_d^T R_3 A_d \\ &+ \frac{d_2^2 - d_1^2}{2} A_d^T R_4 A_d, \\ \Pi_{3,4} &= -N_{33}^T - N_{24} + N_{34}, \\ \Pi_{3,5} &= -N_{31}^T, \\ \Pi_{3,6} &= -N_{33}^T, \\ \Pi_{4,4} &= -Q_2 - N_{34}, \\ \Pi_{4,5} &= -N_{41}^T, \\ \Pi_{4,6} &= -N_{3,4}^T, \\ \Pi_{4,7} &= -N_{24}^T, \\ \Pi_{5,5} &= -\frac{1}{d_1} R_1, \\ \Pi_{6,6} &= -\frac{1}{d_2} R_2, \\ \Pi_{7,7} &= -\frac{1}{d_1} R_2, \\ \Pi_{8,8} &= -\frac{2}{d_1^2} R_3, \\ \Pi_{9,9} &= -\frac{2}{d_2^2 - d_m^2} R_4 \\ \end{split}$$

**Proof:** In order to prove the required result, we define the Lyapunov functional candidate in the following form

$$V(t, x(t)) = \sum_{i=1}^{4} V_i(t, x(t)), \qquad (3.2)$$

where

$$\begin{aligned} V_{1}(t,x(t)) &= x^{T}(t)Px(t), \\ V_{2}(t,x(t)) &= \int_{t-d_{1}}^{t} x^{T}(s)Q_{1}x(s)ds + \int_{t-d_{2}}^{t} x^{T}(s)Q_{2}x(s)ds + \int_{t-d(t)}^{t} x^{T}(s)Q_{3}x(s)ds, \\ V_{3}(t,x(t)) &= d_{1}\int_{d_{1}}^{0}\int_{t+\theta}^{t} \dot{x}^{T}(s)R_{1}\dot{x}(s)dsd\theta + (d_{2}-d_{1})\int_{-d_{2}}^{-d_{1}}\int_{t+\theta}^{t} \dot{x}^{T}(s)R_{2}\dot{x}(s)dsd\theta, \\ V_{4}(t,x(t)) &= \int_{-d_{1}}^{0}\int_{\beta}^{0}\int_{t+\theta}^{t} \dot{x}^{T}(s)R_{3}\dot{x}(s)dsd\theta d\beta + \int_{-d_{2}}^{-d_{1}}\int_{\beta}^{0}\int_{t+\theta}^{t} \dot{x}^{T}(s)R_{4}\dot{x}(s)dsd\theta d\beta, \end{aligned}$$

Calculating the time derivatives of Lyapunov-Krasovskii functional, we have

$$\dot{V}_1(t, x(t)) = 2x^T(t)P\dot{x}(t),$$
(3.3)

$$\dot{V}_{2}(t,x(t)) = x^{T}(t)Q_{1}x(t) - x^{T}(t-d_{1})Q_{1}x(t-d_{1}) + x^{T}(t)[Q_{2}+Q_{3}]x(t) -x^{T}(t-d_{2})Q_{2}x(t-d_{2}) - (1-\mu)x^{T}(t-d(t))Q_{3}x(t-d(t)), \qquad (3.4)$$

$$\dot{V}_{3}(t,x(t)) = \dot{x}^{T}(t) \left[ d_{1}R_{1} + (d_{2} - d_{1})R_{2} \right] \dot{x}(t) - d_{1} \int_{t-d_{1}}^{t} \dot{x}^{T}(s)R_{1}\dot{x}(s)ds - (d_{2} - d_{1}) \int_{t-d_{2}}^{t-d_{1}} \dot{x}^{T}(s)R_{2}\dot{x}(s)ds,$$
(3.5)

$$\dot{V}_{4}(t,x(t)) = \dot{x}^{T}(t) \left[ \left( \frac{d_{1}^{2}}{2} \right) R_{3} + \frac{d_{2}^{2} - d_{1}^{2}}{2} R_{3} \right] \dot{x}(t) - \int_{-d_{1}}^{0} \int_{t+\beta}^{t} \dot{x}^{T}(s) R_{3} \dot{x}(s) ds d\beta - \int_{-d_{2}}^{-d_{1}} \int_{t+\beta}^{t} \dot{x}^{T}(s) R_{4} \dot{x}(s) ds d\beta.$$
(3.6)

By applying Lemma (2.2), the integrals in the equations (3.5), and (3.6) can be written as

$$\begin{split} &-\int_{t-d_{1}}^{t} \dot{x}^{T}(s) R_{1} \dot{x}(s) ds &\leq -\frac{1}{d_{1}} \left( \int_{t-d_{1}}^{t} \dot{x}(s) ds \right)^{T} R_{1} \left( \int_{t-d_{1}}^{t} \dot{x}(s) ds \right), \\ &-\int_{t-d_{2}}^{t-d_{1}} \dot{x}^{T}(s) R_{2} \dot{x}(s) ds &\leq -\frac{1}{d_{2} - d_{1}} \left( \int_{t-d_{2}}^{t-d_{1}} \dot{x}(s) ds \right)^{T} R_{2} \left( \int_{t-d_{1}}^{t} \dot{x}(s) ds \right), \\ &-\int_{-d_{1}}^{0} \int_{t+\theta}^{t} \dot{x}^{T}(s) R_{3} \dot{x}(s) ds d\theta &\leq -\frac{2}{d_{1}^{2}} \left( \int_{-d_{1}}^{t} \int_{t+\theta}^{t} \dot{x}(s) ds d\theta \right)^{T} R_{3} \left( \int_{-d_{1}}^{t} \int_{t+\theta}^{t} \dot{x}(s) ds d\theta \right), \\ &-\int_{t-d_{2}}^{-d_{1}} \int_{t+\theta}^{t} \dot{x}^{T}(s) R_{2} \dot{x}(s) ds d\theta &\leq -\frac{2}{t-d_{2}^{2} - d_{1}^{2}} \left( \int_{t-d_{2}}^{t-d_{1}} \int_{t+\theta}^{t} \dot{x}(s) ds d\theta \right)^{T} R_{4} \\ &\qquad \left( \int_{t-d_{2}}^{t-d_{1}} \int_{t+\theta}^{t} \dot{x}(s) ds d\theta \right). \end{split}$$

On the other hand, by the Newton-Leibniz formula, for any arbitrary matrices  $N_k = [N_{k1}^T \ N_{k2}^T \ N_{k3}^T \ N_{k4}^T]^T$ , k = 1, 2, 3 with compatible dimensions, we have

$$2\zeta^{T}(t)N_{1}\left[x(t) - x(t - d_{1}) - \int_{t - d_{1}}^{t} \dot{x}(s)ds\right] = 0,$$
(3.7)

$$2\zeta^{T}(t)N_{2}\left[x(t-d_{1})-x(t-d(t))-\int_{t-d(t)}^{t-d_{1}}\dot{x}(s)ds\right]=0,$$
(3.8)

$$2\zeta^{T}(t)N_{3}\left[x(t-d_{2})-x(t-d(t))-\int_{t-d(t)}^{t-d_{2}}\dot{x}(s)ds\right]=0,$$
(3.9)

where

$$\zeta^{T}(t) = \left[ x^{T}(t) \ x^{T}(t-d_{1}) \ x^{T}(t-d(t)) \ x^{T}(t-d_{2}) \right]$$

From (3.2)-(3.9), we get,

$$\dot{V}(t, x(t)) \le \alpha^T(t) [\Pi]_{9 \times 9} \alpha(t)$$

where

$$\alpha^{T}(t) = \left[ x^{T}(t) \ x^{T}(t-d_{1}) \ x^{T}(t-d_{1}) \ x^{T}(t-d_{2}) \ \int_{t-d_{1}}^{t} \dot{x}(s)ds \ \int_{t-d(t)}^{t-d_{1}} \dot{x}(s)ds \right]$$
$$\int_{t-d(t)}^{t-d_{2}} \dot{x}(s)ds \ \int_{-d_{1}}^{t} \int_{t+\theta}^{t} \dot{x}(s)dsd\theta \ \int_{t-d_{2}}^{t-d_{1}} \int_{t+\theta}^{t} \dot{x}(s)dsd\theta \right]$$

If the LMI (3.1) holds, we can get  $\dot{V}(t, x(t)) \leq 0$ . Hence, From the definition 2.1, it is concluded that the system (2.1) is asymptotically stable.

Now, we consider the following closed-loop system

$$\dot{x}(t) = (A + BK)x(t) + A_d x(t - d(t)),$$

$$x(t) = \phi(t), \quad \forall t \in [-d_2, 0],$$
(3.10)

In the following theorem, the stability condition for the closed-loop system (3.10) with timevarying delay will be presented based on Theorem 3.1.

**Theorem 3.2.** Consider system (2.1) with the constants  $\mu > 0$ , there exists a feedback control law in the form of equation (2.3) such that the resulting closed-loop form of system (3.10) is asymptotically stable, if there exists matrices X > 0,  $\hat{Q}_i > 0$ ,  $V_k > 0$ , i = 1, 2, 3, k =

1,2,3,4, any real matrices  $\hat{L}_{a,b}$ , a,b = 1,2,3,4 with appropriate dimensions such that the following LMI hold;

$$\left[\Phi\right]_{12\times 12} < 0, \tag{3.11}$$

where,

$$\Phi_{1,1} = 2AX + \hat{Q}_1 + \hat{Q}_2 + \hat{Q}_3 + 2\hat{N}_{11}^T, \\ \Phi_{1,2} = \hat{N}_{11}^T + \hat{N}_{12}^T + \hat{N}_{12}, \\ \Phi_{1,3} = A_dX + \hat{N}_{13} + \hat{N}_{12}^T, \\ + \hat{N}_{1,2}^T, \ \Phi_{1,4} = \hat{N}_{13}^T + \hat{N}_{1,4}^T, \\ \Phi_{1,5} = -\hat{N}_{1,4}, \\ \Phi_{1,6} = -\hat{N}_{31}, \\ \Phi_{1,7} = -\hat{N}_{21},$$

$$\Phi_{1,10} = \sqrt{d_1} (AX + BY)^T \Phi_{1,11} = \sqrt{d_2 - d_1} (AX + BY)^T \Phi_{1,12} = \sqrt{\frac{d_1^2}{2}} (AX + BY)^T \Phi_{1,12}$$

$$\begin{split} \Phi_{1,13} &= \sqrt{\frac{d_2^2 - d_1^2}{2}} (AX + BY)^T, \\ \Phi_{2,2} &= -\hat{Q}_1 - \hat{N}_{1,2} - \hat{N}_{22}, \\ \Phi_{2,3} &= -\hat{N}_{22}^T + \hat{N}_{23}^T - \hat{N}_{13} \\ &+ \hat{N}_{23}, \\ \Phi_{2,4} &= \hat{N}_{2,3}^T - \hat{N}_{14} + \hat{N}_{24}, \\ \Phi_{2,5} &= -\hat{N}_{21}^T, \\ \Phi_{2,6} &= -\hat{N}_{23}^T, \\ \Phi_{2,7} &= -\hat{N}_{23}^T, \\ \Phi_{3,3} &= -(1 - \mu)\hat{Q}_3 + \hat{N}_{23} + \hat{N}_{33}, \\ \Phi_{3,4} &= -\hat{N}_{33}^T - \hat{N}_{24} + \hat{N}_{34}, \\ \Phi_{3,5} &= -\hat{N}_{31}^T, \\ \Phi_{3,6} &= -\hat{N}_{33}^T, \\ \Phi_{3,10} &= \sqrt{d_1} (XA_d)^T \\ \Phi_{3,11} &= \sqrt{d_2 - d_1} (XA_d)^T \\ \Phi_{3,12} &= \sqrt{\frac{d_1^2}{2}} (XA_d)^T \end{split}$$

$$\Phi_{3,13} = \sqrt{\frac{d_2^2 - d_1^2}{2}} (XA_d)^T \Phi_{4,4} = -\hat{Q}_2 - \hat{N}_{34}, \\ \Phi_{4,5} = -\hat{N}_{41}^T, \\ \Phi_{4,6} = -\hat{N}_{3,4}^T, \\ \Phi_{4,7} = -\hat{N}_{24}^T, \\ \Phi_{5,5} = -\frac{1}{d_1} V_1 - 2X, \\ \Phi_{6,6} = -\frac{1}{d_2} V_2 - 2X, \\ \Phi_{7,7} = -\frac{1}{d_1} V_2 - 2X, \\ \Pi_{8,8} = -\frac{2}{d_1^2} V_3 - 2X, \\ \Phi_{9,9} = -\frac{2}{d_2^2 - d_m^2} V_4 - 2X, \\ \Phi_{10,10} = -\sqrt{d_1} V_1, \\ \Phi_{11,11} = -\sqrt{d_1} V_2, \\ \Phi_{12,12} = -\sqrt{\frac{d_1^2}{2}} V_3, \\ \Pi_{10,10} = -\sqrt{d_1} V_1, \\ \Phi_{11,11} = -\sqrt{d_1} V_2, \\ \Phi_{12,12} = -\sqrt{\frac{d_1^2}{2}} V_3, \\ \Pi_{10,10} = -\sqrt{d_1} V_1, \\ \Phi_{11,11} = -\sqrt{d_1} V_2, \\ \Phi_{12,12} = -\sqrt{\frac{d_1^2}{2}} V_3, \\ \Pi_{10,10} = -\sqrt{d_1} V_1, \\ \Phi_{11,11} = -\sqrt{d_1} V_2, \\ \Phi_{12,12} = -\sqrt{\frac{d_1^2}{2}} V_3, \\ \Pi_{10,10} = -\sqrt{d_1} V_1, \\ \Phi_{11,11} = -\sqrt{d_1} V_2, \\ \Phi_{12,12} = -\sqrt{\frac{d_1^2}{2}} V_3, \\ \Pi_{10,10} = -\sqrt{d_1} V_1, \\ \Phi_{11,11} = -\sqrt{d_1} V_2, \\ \Phi_{12,12} = -\sqrt{\frac{d_1^2}{2}} V_3, \\ \Pi_{10,10} = -\sqrt{d_1} V_1, \\ \Phi_{11,11} = -\sqrt{d_1} V_2, \\ \Phi_{12,12} = -\sqrt{\frac{d_1^2}{2}} V_3, \\ \Pi_{10,10} = -\sqrt{d_1} V_1, \\ \Phi_{11,11} = -\sqrt{d_1} V_2, \\ \Phi_{12,12} = -\sqrt{\frac{d_1^2}{2}} V_3, \\ \Pi_{10,10} = -\sqrt{d_1} V_1, \\ \Phi_{11,11} = -\sqrt{d_1} V_2, \\ \Phi_{12,12} = -\sqrt{\frac{d_1^2}{2}} V_3, \\ \Pi_{10,10} = -\sqrt{d_1} V_1, \\ \Phi_{11,11} = -\sqrt{d_1} V_2, \\ \Phi_{12,12} = -\sqrt{\frac{d_1^2}{2}} V_3, \\ \Pi_{10,10} = -\sqrt{d_1} V_1, \\ \Phi_{11,11} = -\sqrt{d_1} V_2, \\ \Phi_{12,12} = -\sqrt{\frac{d_1^2}{2}} V_3, \\ \Pi_{10,10} = -\sqrt{d_1} V_1, \\ \Phi_{11,11} = -\sqrt{d_1} V_2, \\ \Phi_{12,12} = -\sqrt{\frac{d_1^2}{2}} V_3, \\ \Psi_{11,11} = -\sqrt{d_1} V_1, \\ \Psi_{11,11} = -\sqrt{d_1} V_1$$

$$\Phi_{13,13} = -\sqrt{\frac{d_2^2 - d_1^2}{2}}V_4$$

Moreover, the gain matrix of the feedback reliable controller (2.3) can be obtained by  $K = YX^{-1}$ .

**Proof:** The proof of this theorem follows immediately from the Theorem (3.1). In order to obtain the feedback controller gain matrices, take  $\overline{W} = \{X, X, ...X\} \in \mathbb{R}^{9 \times 9}$ . Pre and post multiplying (3.1) by diag  $\{\overline{W}, I, I, I, I\}$ , where  $X = P^{-1}, V_1 = R_1^{-1}, V_2 = R_2^{-1}, V_3 = R_3^{-1}$  and  $V_4 = R_4^{-1}$ . Let  $\hat{Q}_i = XQ_iX$ , i=1,2,3,  $\hat{N}_k = XL_kX$ , k = 1, 2, 3, and  $K = YX^{-1}$ , using the inequalities  $-XV_i^{-1}X \leq V_i - 2X$ , i = 1, 2, 3, 4, we obtain the LMI (3.11). Then based on the Theorem (3.1) and Definition (2.1), it is concluded that the considered nominal closed-loop system (3.10) is asymptotically stable. The proof is completed.

#### **4** Numerical Example

Consider the system (2.1) with following parameters

$$A = \begin{bmatrix} -0.2 & 1.2 & -1.4 \\ 2.3 & 0.1 & -0.5 \\ 0.4 & 0.2 & -0.8 \end{bmatrix}, A_d = \begin{bmatrix} -0.2 & 1.2 & -1.4 \\ 2.3 & 0.1 & -0.5 \\ 0.4 & 0.2 & -0.8 \end{bmatrix}, B = \begin{bmatrix} 0, 1 \\ 0.2 \\ -0.1 \end{bmatrix}$$
(4.1)

By solving the LMI in Theorem (3.1) based, the calculated maximum upper bound  $d_2$  for different values of  $d_1$  with  $\mu = 0.5$  is provided in Table 1.

Solving the LMI in theorem 3.2, we get the feasible solution when  $d_1 = 0.3$ ,  $d_2 = 0.35$  and  $\mu = 0.5$  and the controller gain  $K = [-142.3318 - 117.1139 \ 119.7970]$ .



**Table 1.** Calculated maximum upper bound for various values of  $d_1$ 

0.5

0.9655

0.7

1.1062

0.9

1.2572

0.3

0.8369

Figure 1. State responses without control

0.1

0.7315

 $d_1$ 

do

Figure 2. State responses with control

Figs.1-2 represents the simulation result for trajectories of system with and with out control gain with condition  $[-0.2 \ 0.3 \ 0.2]$  respectively. Figure 2 is plotted by using the controller gain K value. It is concluded from the simulation results that the state trajectories of the system without control do not converge to equilibrium point but state trajectories converges to equilibrium point quickly with the proposed controller which demonstrates the applicability of our controller design method.

# 5 Conclusion

In this paper, stability and stabilization problem for a continuous time systems with time varying delay is studied. By implementing the Lyapunov approach, a new set of condition is obtained which ensures that the system is asymptotically stable. In particular, the feedback control law is designed in terms of the solution of certain linear matrix inequality. More precisely, the solvability of the addressed problem has been expressed as the feasibility of a set of LMI. Finally, we have provided numerical study with an example to validate the effectiveness of the proposed design techniques.

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Received : January 4, 2021 Accepted : April 15, 2021