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ON r-FUZZY WEAKLY b-OPEN FUNCTIONS

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Abstract In this paper, we introduce and characterize a new class of functions called r-fuzzy weakly b-open (r-fuzzy weakly b-closed) functions between smooth fuzzy topological spaces.

1 Introduction

The fuzzy concept has invaded almost all branches of Mathematics since its introduction by Zadeh [20]. Fuzzy sets have applications in many fields such as information [15] and control [18]. The theory of fuzzy topological spaces was introduced and developed by Chang [1] and since then various notions in classical topology have been extended to fuzzy topological spaces. S ǒstak [16] and Kubiak [8] introduced the fuzzy topology as an extension of ChangâĂŹs fuzzy topology. It has been developed in many directions. S ǒstak [17] also published a survey article of the developed areas of fuzzy topological spaces. In this paper, we introduce and characterize a new class of functions called r-fuzzy weakly b-open (r-fuzzy weakly b-closed) functions between smooth fuzzy topological spaces.

Definition 1.1. A fuzzy point x_t in X is a fuzzy set taking value $t \in I_0$ at x and zero elsewhere, $x_t \in \lambda$ if and only if $t \leq \lambda(x)$. A fuzzy set λ is quasicoincident with a fuzzy set μ, denoted by $\lambda q \mu$, if there exists $x \in X$ such that $\lambda(x) + \mu(x) > 1$. Otherwise $\lambda \bar{q} \mu$.

Definition 1.2. [8, 16] A function $\tau : I^X \to I$ is called a smooth fuzzy topology on X if it satisfies the following conditions:

(i) $\tau(\bar{0}) = \tau(\bar{1}) = 1$;

(ii) $\tau(\mu_1 \wedge \mu_2) \geq \tau(\mu_1) \wedge \tau(\mu_2)$ for any $\mu_1, \mu_2 \in I^X$.

(iii) $\tau(\bigvee_{j\in\Gamma} \mu_j) \geq \bigvee_{j\in\Gamma} \tau(\mu_j)$ for any $\{\mu_j\}_{j\in\Gamma} \in I^X$.
The pair (X, τ) is called a smooth fuzzy topological space.

A fuzzy point in X with support $x \in X$ and the value $\alpha (0 < \alpha \leq 1)$ is denoted by x_α.

Definition 1.3. [11] A fuzzy set $\lambda \in I^X$ is said to be q-coincident with a fuzzy set μ, denoted by $\lambda q \mu$, if there exists $x \in X$ such that $\lambda(x) + \mu(x) > 1$. It is known that $\lambda \leq \mu$ if and only if λ and $1 - \mu$ are not q-coincident, denoted by $\lambda \bar{q} (1 - \mu)$.

Definition 1.4. [3] A fuzzy set λ is said to be r-fuzzy Q-neighbourhood of x_p if $\tau(\lambda) \geq r$ such that $x_p q \lambda$. We will denote the set of all r-fuzzy open Q-neighbourhood of x_p by $\mathcal{Q}(x_p, r)$.

Definition 1.5. [2] Let (X, τ) be a smooth fuzzy topological space. For each $\lambda \in I^X$, $r \in I_0$, an operator $Cl : I^X \times I_0 \to I^X$ is defined as follows: $Cl(\lambda, r) = \wedge\{\mu : \mu \geq \lambda, \tau(\bar{1} - \mu) \geq r\}$. For $\lambda, \mu \in I^X$ and $r, s \in I_0$, it satisfies the following conditions:

(i) $Cl(\bar{0}, r) = \bar{0}$.

(ii) $\lambda \leq Cl(\lambda, r)$.

(iii) $Cl(\lambda, r) \vee Cl(\mu, r) = Cl(\lambda \vee \mu, r)$.

(iv) $Cl(\lambda, r) \leq Cl(\lambda, s)$ if $r \leq s$.

(v) $Cl(Cl(\lambda, r), r) = Cl(\lambda, r)$.

Proposition 1.6. *[12] Let (X, τ) be a smooth fuzzy topological space. For each $\lambda \in I^X$, $r \in I_0$, an operator $Int : I^X \times I_0 \to I^X$ is defined as follows: $Int(\lambda, r) = \vee\{\mu : \mu \leq \lambda, \tau(\mu) \geq r\}$. For $\lambda, \mu \in I^X$ and $r, s \in I_0$, it satisfies the following conditions:*

(i) $Int(\bar{1} - \lambda, r) = \bar{1} - Cl(\lambda, r)$.

(ii) $Int(\bar{1}, r) = \bar{1}$.

(iii) $\lambda \geq Int(\lambda, r)$.

(iv) $Int(\lambda, r) \wedge Int(\mu, r) = Int(\lambda \wedge \mu, r)$.

(v) $Int(\lambda, r) \geq Int(\lambda, s)$, *if* $r \leq s$.

(vi) $Int(Int(\lambda, r), r) = Int(\lambda, r)$.

Definition 1.7. [10] A fuzzy point x_p is said to be a r-fuzzy θ-cluster point of a fuzzy set λ if and only if for every $\mu \in \mathcal{Q}(x_p, r)$, $Cl(\mu, r)$ is q-coincident with λ. The set of all r-fuzzy θ-cluster points of λ is called the r-fuzzy θ-closure of λ and will be denoted by $Cl_\theta(\lambda, r)$. A fuzzy set λ will be called r-fuzzy θ-closed if and only if $\lambda = Cl_\theta(\lambda, r)$. The complement of a r-fuzzy θ-closed set is called r-fuzzy θ-open and the r-fuzzy θ-interior of λ denoted by $Int_\theta(\lambda, r)$ is defined as $Int_\theta(\lambda, r) = \{x_p$: for some $\beta \in \mathcal{Q}(x_p, r)$, $Cl(\beta, r) \leq \lambda\}$.

Lemma 1.8. *[10] Let λ be a fuzzy set in a smooth fuzzy topological space (X, τ). Then,*

(i) λ is r-fuzzy θ-open if, and only if $\lambda = Int_\theta(\lambda, r)$;

(ii) $1 - Int_\theta(\lambda, r) = Cl_\theta(1 - \lambda, r)$ *and* $Int_\theta(1 - \lambda, r) = 1 - Cl_\theta(\lambda, r)$;

(iii) $Cl_\theta(\lambda, r)$ is a r-fuzzy closed set but not necessarily is a r-fuzzy θ-closed set.

Definition 1.9. A fuzzy set λ of a smooth fuzzy topological space (X, τ) is called:

(i) r-fuzzy preopen [12] if $\lambda \leq Int(Cl(\lambda, r), r)$;

(ii) r-fuzzy regular open [6] if $\lambda = Int(Cl(\lambda, r), r)$;

(iii) r-fuzzy α-open [12] if $\lambda \leq Int(Cl(Int(\lambda, r), r), r)$;

(iv) r-fuzzy preclosed [12] if $Cl(Int(\lambda, r), r) \leq \lambda$;

(v) r-fuzzy regular closed [6] if $\lambda = Cl(Int(\lambda, r), r)$;

(vi) r-fuzzy α-closed [12] if $Cl(Int(Cl(\lambda, r), r), r) \leq \lambda$;

(vii) r-fuzzy b-open if $\lambda \leq Cl(Int(\lambda, r), r) \vee Int(Cl(\lambda, r), r)$;

(viii) r-fuzzy b-closed if $Cl(Int(\lambda, r), r) \wedge Int(Cl(\lambda, r), r) \leq \lambda$.

Definition 1.10. Let (X, τ) be a smooth fuzzy topological space. For each $\lambda \in I^X$, $r \in I_0$, an operator $bCl : I^X \times I_0 \to I^X$ is defined as $bCl(\lambda, r) = \wedge\{\mu : \mu \geq \lambda, \mu$ is r-fuzzy b-closed$\}$. An operator $bInt : I^X \times I_0 \to I^X$ is defined as $bInt(\lambda, r) = \vee\{\mu : \mu \leq \lambda, \mu$ is r-fuzzy b-open$\}$.

Definition 1.11. A function $f : (X, \tau) \to (Y, \sigma)$ is called:

(i) r-fuzzy b-open if $f(\lambda)$ is a r-fuzzy b-open set in Y for each fuzzy open set λ of X;

(ii) r-fuzzy weakly open if $f(\lambda) \leq Int(f(Cl(\lambda, r)), r)$ for each fuzzy open set λ of X.

2 r-Fuzzy weakly b-open functions

Definition 2.1. A function $f : (X, \tau) \to (Y, \sigma)$ is said to be r-fuzzy weakly b-open if $f(\lambda) \leq bInt(f(Cl(\lambda, r)), r)$ for each $\tau(\lambda) \geq r$.

Remark 2.2. It is evident that, every r-fuzzy weakly open function is r-fuzzy weakly b-open and every r-fuzzy b-open function is also r-fuzzy weakly b-open. But the converse need not be true in general.

Example 2.3. Let $X = \{a, b, c\}$ and $\mu = \left(\frac{a}{0.2}, \frac{b}{0.3}, \frac{c}{0.4}\right)$, and $\beta = \left(\frac{a}{0.2}, \frac{b}{0.3}, \frac{c}{0.3}\right)$. Define $\tau : L^X \to L$ and $\sigma : L^Y \to L$ as follows:

$$\tau(\lambda) = \begin{cases} 1 & \text{if } \lambda = \overline{0} \text{ or } \overline{1} \\ \frac{1}{2} & \text{if } \lambda = \mu \\ 0 & otherwise \end{cases} \qquad \sigma(\lambda) = \begin{cases} 1 & \text{if } \lambda = \overline{0} \text{ or } \overline{1} \\ \frac{1}{2} & \text{if } \lambda = \beta \\ 0 & otherwise. \end{cases}$$

Then the identity function $f : (X, \tau) \to (Y, \sigma)$ defined by $g(a) = c$, $g(b) = a$ and $g(c) = b$ is $\frac{1}{2}$-fuzzy weakly b-open but not $\frac{1}{2}$-fuzzy b-open.

Definition 2.4. A smooth fuzzy topological space (X, τ) is r-fuzzy regular if for each $x \in X$, $\rho \in [0, 1]$, $\tau(\lambda) \geq r$ with $x_p \leq \lambda$, there exists $\tau(\mu) \geq r$ such that $x_p \leq Cl(\mu, r) \leq \lambda$.

Theorem 2.5. *For a surjective function $f : (X, \tau) \to (Y, \sigma)$, the following conditions are equivalent:*

(i) f is r-fuzzy weakly b-open;

(ii) $f(Int_\theta(\lambda, r)) \leq bInt(f(\lambda), r)$ for every $\lambda \in I^X$;

(iii) $Int_\theta(f^{-1}(\beta), r) \leq f^{-1}(bInt(\beta, r))$ for every $\beta \in I^Y$;

(iv) $f^{-1}(bCl(\beta, r)) \leq Cl_\theta(f^{-1}(\beta), r)$ for every $\beta \in I^Y$;

(v) For each r-fuzzy θ-open set $\lambda \in I^X$, $f(\lambda)$ is r-fuzzy b-open in Y;

(vi) For any $\beta \in I^Y$ and any r-fuzzy θ-closed set $\lambda \in I^X$ containing $f^{-1}(\beta)$, where X is a r-fuzzy regular space, there exists a r-fuzzy b-closed $\delta \in I^Y$ containing β such that $f^{-1}(\delta) \leq \lambda$.

Proof. (1)⇔(2): Let $\lambda \in I^X$ and x_p be a fuzzy point in $Int_\theta(\lambda, r)$. Then there exists a fuzzy open q-neighbourhood μ of x_p such that $\mu \leq Cl(\mu, r) \leq \lambda$. Then $f(\mu) \leq f(Cl(\mu, r)) \leq f(\lambda)$. Since f is r-fuzzy weakly b-open, $f(\mu) \leq bInt(f(Cl(\mu, r)), r) \leq bInt(f(\lambda), r)$. Then $f(x_p)$ is a fuzzy point in $bInt(f(\lambda), r)$. Hence $x_p \in f^{-1}(bInt(f(\lambda), r))$. Thus $Int_\theta(\lambda, r) \leq f^{-1}(bInt(f(\lambda), r))$, and so $f(Int(\lambda), r) \leq bInt(f(\lambda), r)$. Conversely, let $\tau(\mu) \geq r$. Since $\mu \leq Int_\theta(Cl(\mu, r), r)$, we have $f(\mu) \leq f(Int_\theta(Cl(\mu, r), r)) \leq bInt(f(Cl(\mu, r)), r)$. Hence f is r-fuzzy weakly b-open.
(2)⇔(3): Let $\beta \in I^Y$. Then by (2), $f(Int_\theta(f^{-1}(\beta), r)) \leq bInt(\beta, r)$. Therefore, $Int_\theta(f^{-1}(\beta), r) \leq f^{-1}(bInt(\beta, r))$. The converse is clear.
(3)⇔(4): Let $\beta \in I^Y$. By (3), $\overline{1} - Cl_\theta(f^{-1}(\beta), r) = Int_\theta(\overline{1} - f^{-1}(\beta), r) = Int_\theta(f^{-1}(\overline{1} - \beta), r) \leq f^{-1}(bInt(\overline{1} - \beta, r)) = f^{-1}(\overline{1} - bCl(\beta, r)) = \overline{1} - f^{-1}(bCl(\beta, r))$. Therefore, $f^{-1}(bCl(\beta, r)) \leq Cl_\theta(f^{-1}(\beta), r)$. The converse is clear.
(4)⇒(5): Let λ be a r-fuzzy θ-open set in X. Then $\overline{1} - f(\lambda) \in I^Y$ and by (4), $f^{-1}(bCl(\overline{1} - f(\lambda)), r) \leq Cl_\theta(f^{-1}(\overline{1} - f(\lambda)), r)$. Therefore, $\overline{1} - f^{-1}(bInt(f(\lambda), r)) \leq Cl_\theta(\overline{1} - \lambda, r) = \overline{1} - \lambda$. Then we have $\lambda \leq f^{-1}(bInt(f(\lambda), r))$ which implies $f(\lambda) \leq bInt(f(\lambda), r)$. Hence $f(\lambda)$ is r-fuzzy b-open in Y.
(5)⇒(6): Let $\beta \in I^Y$ and $\lambda \in I^X$ be a r-fuzzy θ-closed set such that $f^{-1}(\beta) \leq \lambda$. Since $\overline{1} - \lambda$ is r-fuzzy θ-open in X, by (5), $f(\overline{1} - \lambda)$ is r-fuzzy b-open in Y. Let $\delta = \overline{1} - f(\overline{1} - \lambda)$. Then δ is r-fuzzy b-closed and $\beta \leq \delta$. Now, $f^{-1}(\delta) = f^{-1}(\overline{1} - (f(\overline{1} - \lambda))) = \overline{1} - f^{-1}(f(\lambda)) \leq \lambda$.
(6)⇒(4): Let $\beta \in I^Y$. Then by Corollary 3.6 of [**?**] $\lambda = Cl_\theta(f^{-1}(\beta), r)$ is r-fuzzy θ-closed in X and $f^{-1}(\beta) \leq \lambda$. Then there exists a r-fuzzy b-closed set $\delta \in Y^X$ containing β such that $f^{-1}(\delta) \leq \lambda$. Since δ is r-fuzzy b-closed, $f^{-1}(bCl(\beta, r) \leq f^{-1}(\delta) \leq Cl_\theta(f^{-1}(\beta), r)$. □

Theorem 2.6. *A function $f : (X, \tau) \to (Y, \sigma)$ is r-fuzzy weakly b-open if, and only if for each fuzzy point x_p in X and each $\tau(\mu) \geq r$ containing x_p, there exists a r-fuzzy b-open set δ containing $f(x_p)$ such that $\delta \leq f(Cl(\mu, r))$.*

Proof. Let $x_p \in X$ and $\tau(\mu) \geq r$ such that μ containing x_p. Since f is r-fuzzy weakly b-open, $f(\mu) \leq bInt(f(Cl(\mu, r)), r)$. Let $\delta = bInt(f(Cl(\mu, r)), r)$. Hence $\delta \leq f(Cl(\mu, r))$ with δ containing $f(x_p)$. Conversely, let $\tau(\mu) \geq r$ and let $y_p \in f(\mu)$. Then $\delta \leq f(Cl(\mu, r))$ for some r-fuzzy b-open set δ in Y containing y_p. Hence we have, $y_p \in \delta \leq bInt(f(Cl(\mu, r)), r)$. Hence $f(\mu) \leq bInt(f(Cl(\mu, r)), r)$ and f is r-fuzzy weakly b-open. □

Theorem 2.7. *Let $f : (X, \tau) \to (Y, \sigma)$ be a bijective function. Then the following statements are equivalent:*

(i) f is r-fuzzy weakly b-open;

(ii) $bCl(f(\lambda), r) \leq f(Cl(\lambda, r))$ for each $\tau(\lambda) \geq r$;

(iii) $bCl(f(Int(\beta, r)), r) \leq f(\beta)$ for each $\tau(\overline{1} - \beta) \geq r$.

Proof. (1)⇔(3): Let $\tau(\overline{1} - \beta) \geq r$. Then $f(\overline{1} - \beta) = \overline{1} - f(\beta) \leq bInt(f(Cl(\overline{1} - \beta, r)), r)$ and so $\overline{1} - f(\beta) \leq \overline{1} - bCl(f(Int(\beta, r)), r)$. Hence $bCl(f(Int(\beta, r)), r) \leq f(\beta)$. The converse is clear. (3)⇔(2): Let $\tau(\lambda) \geq r$. Since $Cl(\lambda, r)$ is a r-fuzzy closed and $\lambda \leq Int(Cl(\lambda, r), r)$ by (3), $bCl(f(\lambda), r) \leq bCl(f(Int(Cl(\lambda, r), r)), r) \leq f(Cl(\lambda, r))$. The converse is clear. □

The proof of the following theorem is obvious and thus omitted.

Theorem 2.8. *For a function $f : (X, \tau) \to (Y, \sigma)$ the following statements are equivalent:*

(i) f is r-fuzzy weakly b-open;

(ii) for each $\tau(\overline{1} - \beta) \geq r$, $f(Int(\beta, r)) \leq bInt(f(\beta), r)$;

(iii) for each $\tau(\lambda) \geq r$, $f(Int(Cl(\lambda, r), r)) \leq bInt(f(Cl(\lambda, r)), r)$;

(iv) for each r-fuzzy regular open set $\lambda \in I^X$, $f(\lambda) \leq bInt(f(Cl(\lambda, r)), r)$;

(v) for every r-fuzzy preopen set $\lambda \in I^X$, $f(\lambda) \leq bInt(f(Cl(\lambda, r)), r)$;

(vi) for every r-fuzzy α-open set $\lambda \in I^X$, $f(\lambda) \leq bInt(f(Cl(\lambda, r)), r)$.

Definition 2.9. A function $f : (X, \tau) \to (Y, \sigma)$ is said to satisfy the r-fuzzy weakly b-open interiority condition of $bInt(f(Cl(\lambda, r)), r) \leq f(\lambda)$ for every $\tau(\lambda) \geq r$.

Theorem 2.10. *If $f : (X, \tau) \to (Y, \sigma)$ is r-fuzzy weakly b-open and satisfies fuzzy weakly b-open interiority condition, then f is fuzzy b-open.*

Proof. Let $\tau(\lambda) \geq r$. Since f is r-fuzzy weakly b-open, we have $f(\lambda) \leq bInt(f(Cl(\lambda, r)), r)$. However, because f satisfies the r-fuzzy weakly b-open interiority condition, $f(\lambda) = bInt(f(Cl(\lambda, r)), r)$ and hence $f(\lambda)$ is r-fuzzy b-open. □

Theorem 2.11. *Let (X, τ) be a r-fuzzy regular space. Then $f : (X, \tau) \to (Y, \sigma)$ is r-fuzzy weakly b-open if and only if f is r-fuzzy b-open.*

Proof. Let $\tau(\lambda) \geq r$ and $\lambda \neq \overline{0}$. For each fuzzy point x_p in λ, let $\tau(\mu_{x_p}) \geq r$ such that $x_p \in \mu_{x_p} \leq Cl(\mu_{x_p}, r) \leq \lambda$. Hence we have $\lambda = \vee\{\mu_{x_p}: x_p \in \lambda\} \vee\{Cl(\mu_{x_p}, r): x_p \in \lambda\}$ and, $f(\lambda) = \vee\{f(\mu_{x_p}): x_p \in \lambda\} \leq \vee\{bInt(f(Cl(\mu_{x_p}, r)), r): x_p \in \lambda\} \leq bInt(f(\vee\{Cl(\mu_{x_p}, r)), r): x_p \in \lambda\})$ $= bInt(f(\lambda), r)$. Thus, f is r-fuzzy b-open. □

Definition 2.12. A function $f : (X, \tau) \to (Y, \sigma)$ is said to be r-fuzzy contra-pre-b-closed provided that $f(\lambda)$ is r-fuzzy b-open for each r-fuzzy b-closed subset $\lambda \in I^X$.

Theorem 2.13. *If $f : (X, \tau) \to (Y, \sigma)$ is r-fuzzy weakly b-open and Y has the property that union of r-fuzzy b-closed sets is r-fuzzy b-closed and if for each r-fuzzy b-closed subset β of X and each fiber $f^{-1}(y_p) \leq \overline{1} - \beta$ there exists $\tau(\mu) \geq r$ for which $\beta \leq \mu$ and $f^{-1}(y_p)\ \bar{q}\ Cl(\mu)$, then f is r-fuzzy contra-pre-b-closed.*

Proof. Assume that $\beta \in I^X$ is r-fuzzy b-closed set and let $y_p \in \overline{1} - f(\beta)$. Thus, $f^{-1}(y_p) \leq \overline{1} - \beta$ and hence there exists $\tau(\mu) \geq r$ for which $\beta \leq \mu$ and $f^{-1}(y_p)\ \bar{q}\ Cl(\mu, r)$. Therefore, $y_p \in \overline{1} - f(Cl(\mu, r)) \leq \overline{1} - f(\beta)$. Since f is r-fuzzy weakly b-open, $f(\mu) \leq bInt(f(Cl(\mu, r)), r)$. Hence $y_p \in bCl(\overline{1} - f(Cl(\mu, r)), r) \leq \overline{1} - f(\beta)$. Let $\lambda_{y_p} = bCl(\overline{1} - f(Cl(\mu, r)), r)$. Then λ_{y_p} is a r-fuzzy b-closed set of Y containing y_p. Hence $\overline{1} - f(\beta) = \vee\{\lambda_{y_p}: y_p \in \overline{1} - f(\beta)\}$ is r-fuzzy b-closed and $f(\beta)$ is r-fuzzy b-open. □

Definition 2.14. Two non-zero fuzzy sets λ and μ in a smooth fuzzy topological space (X, τ) are said to be r-fuzzy b-separated if $\lambda\, \bar{q}\, bCl(\mu, r)$ and $\mu\, \bar{q}\, bCl(\lambda, r)$ or equivalently, if there exist two r-fuzzy b-open sets ρ and η such that $\lambda \leq \rho$ and $\mu \leq \eta$, $\lambda\, \bar{q}\, \eta$ and $\mu\, \bar{q}\, \rho$.

Definition 2.15. A smooth fuzzy topological space (X, τ) which cannot be expressed as the union of two r-fuzzy b-separated sets is said to be a r-fuzzy b-connected space.

Theorem 2.16. *If $f : (X, \tau) \to (Y, \sigma)$ is a r-fuzzy weakly b-open surjective function of a smooth fuzzy topological space (X, τ) to a r-fuzzy b-connected space (Y, σ), then (X, τ) is r-fuzzy connected.*

Proof. If possible, let (X, τ) be not r-fuzzy connected. Then there exist fuzzy separated sets λ and μ in X such that $\overline{1} = \lambda \vee \mu$. Since λ and μ are r-fuzzy separated, there exist $\tau(\rho) \geq r$ and $\tau(\eta) \geq r$ such that $\lambda \leq \rho$, $\mu \leq \eta$, $\lambda\, \bar{q}\, \eta$ and $\mu\, \bar{q}\, \rho$. Hence we have $f(\lambda) \leq f(\rho)$, $f(\mu) \leq f(\eta)$, $f(\lambda)\, \bar{q}\, f(\eta)$ and $f(\mu)\, \bar{q}\, f(\rho)$. Since f is r-fuzzy weakly b-open, $f(\rho) \leq bInt(f(Cl(\rho, r)), r)$ and $f(\eta) \leq bInt(f(Cl(\eta, r)), r)$ and since ρ and η are r-fuzzy open and also r-fuzzy closed, we have $f(Cl(\rho, r)) = f(\rho)$, $f(Cl(\eta, r)) = f(\eta)$. Hence $f(\rho)$ and $f(\eta)$ are r-fuzzy b-open in Y. Therefore, $f(\lambda)$ and $f(\eta)$ are r-fuzzy b-separated sets in Y and $\overline{1} = f(\overline{1}) = f(\lambda \vee \mu) = f(\lambda) \vee f(\mu)$. Hence this contrary to the fact that Y is r-fuzzy b-connected. Thus, X is r-fuzzy connected. □

Definition 2.17. A function $f : (X, \tau) \to (Y, \sigma)$ is said to be fuzzy weakly b-closed if $bCl(f(Int(\lambda, r)), r) \leq f(\lambda)$ for each $\tau(\overline{1} - \lambda) \geq r$.

The proof of the following two Theorems are obvious and hence omitted.

Theorem 2.18. *For a function $f : (X, \tau) \to (Y, \sigma)$, the following conditions are equivalent:*

(i) *f is r-fuzzy weakly b-closed;*

(ii) *$bCl(f(\lambda), r) \leq f(Cl(\lambda, r))$ for every $\tau(\lambda) \geq r$;*

(iii) *$bCl(f(\lambda), r) \leq f(Cl(\lambda), r)$ for every r-fuzzy regular open set $\lambda \in I^X$;*

(iv) *For each $\mu \in I^Y$ and every $\tau(\eta) \geq r$ with $f^{-1}(\mu) \leq \eta$, there exists a r-fuzzy b-open set $\delta \in I^Y$ with $\mu \leq \delta$ and $f^{-1}(\mu) \leq Cl(\eta, r)$;*

(v) *For each fuzzy point y_p in Y and each $\tau(\eta) \geq r$ with $f^{-1}(y_p) \leq \eta$, there exists a r-fuzzy b-open set $\delta \in I^Y$ with $y_p \leq \delta$ and $f^{-1}(\delta) \leq Cl(\eta, r)$;*

(vi) *$bCl(f(Int(Cl(\lambda, r), r)), r) \leq f(Cl(\lambda, r))$ for each $\lambda \in I^X$;*

(vii) *$bCl(f(Int(Cl_\theta(\lambda, r), r)), r) \leq f(Cl_\theta(\lambda, r))$ for each $\lambda \in I^X$;*

(viii) *$bCl(f(\lambda), r) \leq f(Cl(\lambda, r))$ for each r-fuzzy b-open set $\lambda \in I^X$.*

Theorem 2.19. *For a function $f : (X, \tau) \to (Y, \sigma)$, the following conditions are equivalent:*

(i) *f is fuzzy weakly b-closed;*

(ii) *$bCl(f(Int(\lambda, r)), r) \leq f(\lambda)$ for each r-fuzzy b-closed set $\lambda \in I^X$;*

(iii) *$bCl(f(Int(\lambda, r)), r) \leq f(\lambda)$ for each r-fuzzy α-closed set $\lambda \in I^X$.*

Theorem 2.20. *If $f : (X, \tau) \to (Y, \sigma)$ is r-fuzzy weakly b-closed injective, then for each $\lambda \in I^Y$ and each $\tau(\mu) \geq r$ with $f^{-1}(\lambda) \leq \mu$ there exists a r-fuzzy b-closed set $\rho \in I^Y$ with $\lambda \leq \rho$ and $f^{-1}(\rho) \leq Cl(\mu)$.*

Proof. Follows from the definitions. □

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