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V SUPER VERTEX MAGIC LABELING OF SOME FAMILIES OF GRAPHS

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Abstract Let G be a finite simple graph with p vertices and q edges. A vertex magic total labeling is a bijection from $V(G) \cup E(G)$ to the consecutive integers 1,2,3,......,$p+q$ with the property that every $u \in V(G)$, $f(u) + \sum_{v \in N(u)} f(uv) = k$ for some constant K. such a labeling is V-Super if $f[V(G)]$=1,2,3,....,p. A graph G is called V-Super vertex magic if it admits a V-Super vertex magic labeling. In this paper, we establish the V-Super vertex magic labeling of some classes of $H_{2r+1,4n}$ graph, $H_{4,2n+5}$ graph and parity graph

1 Introduction

Graph theory is a main stream of mathematics. In the year of 1736, Leonhard Euler was introduced the subject graph theory, when he began discussing whether or not it was possible to cross all of the bridges in the city of Kaliningrad, Russia only once. Since, the problem has been thoroughly identified by mathematics such as Thomas Ponnyngton Kirkman and William Roman Hamilton find in original problem. The vertices represented locations around the city which were connected by bridges or the edges of the graph. One of the most famous problems in graph theory is the four colors and five colors conjecture proposed in 1850. The vertices and edges of a graph can be labeled in different types of ways. The most famous method is of labeling the vertices with numbers. An interesting area of vertex labeling with numbers is known as vertex-magic. Vertex-magic graphs are graphs labeled with numbers in which every vertex and its incident edges add up to the same number. This number is known as magic number. The field of graph theory plays an important role in various areas of pure and applied sciences. Let G be a finite, undirected graph with no loops and multiple edges. Let G be a graph with vertex set $V(G)$ and edge set $E(G)$.

A magic graph is a graph whose edges are labeled by positive integers, so that the sum over the edges incident with any vertex is the same independent of the choice of vertex. A graph is vertex magic if its vertices can be labeled so that the sum on any edges is the same. It is total magic if its edges and vertices can be labeled so that the vertex, plus the sum of labels on edges incident with that vertex is a constant.

Sedlacek (1963) introduced the concept of magic labeling. Suppose that G is a graph with q edges we say that G is magic if the edges of G can be labeled by the numbers 1,2,3,.....,q so that the sum of labels of all the edges incident with any vertex is same. Macdougall et.al.(2002) introduced the notion of vertex magic total labeling. If G is a finite simple undirected graph with p vertices and q edges, then a vertex magic total labeling is a bijection from $V(G) \cup E(G)$ to the integers 1,2,3,.....,$p+q$ with the property that for every u in $V(G)$.

2 Preliminaries

Definition 2.1: A graph G with p vertices and q edges is called **V-super vertex magic** if there exit a bijection $f : V \to \{1, 2, 3..., p\}$ and $f : E \to \{p+1, p+2, ..., p+q\}$ and for this labeling there is some constant K such that for each vertex p.

Definition 2.2: A **Parity graph** is a graph in which every two induced paths between the same two vertices have the same equality either both paths have odd length, or both have even length.

Theorem 2.1[2] If G has a E-super vertex magic total labelling,then $k = q + \frac{(p+1)}{2} + \frac{q(q+1)}{p}$.

Theorem 2.2[2] If G has a super vertex magic total labelling,then $k = \frac{(p+q)(p+q+1)}{p} - \frac{(p+1)}{2}$.

Lemma 2.1[3] If a non-trivial graph G is an V-super vertex magic total, then the magic constant k is given by $k = 2q + \frac{(p+1)}{2} + \frac{q(q+1)}{p}$.

Lemma 2.2[4] If G is a vertex magic graph with p vertices and q edges, then $\frac{(p+q)(p+q+1)}{2p}+\frac{E_{sum}}{p}$=k.

3 V - Super Vertex Magic Graphs

Theorem 3.1. *The graph $H_{2r+1,4n}$ has a V-super vertex magic total labeling if $r \geq 1$ and $n \geq 2$.*

Proof.

Case:1

Let $H_{2r+1,4n}$ be the graph with each vertex of degree is $2n+1$, if r=1 and then 4n vertices and $6n$ edges.

Suppose $H_{2r+1,4n}$ has a V-super vertex magic total labeling, then by theorem 2.2,

$$k = \frac{(p+q)(p+q+1)}{p} - \frac{p+1}{2}$$

In graph, $p = 4n$ and $q = 6n$

$$\begin{aligned} k &= \frac{(4n+6n)(4n+6n+1)}{4n} - \frac{4n+1}{2} \\ k &= \frac{(10n)(10n+1)}{4n} - \frac{4n+1}{2} \\ k &= 23n+2 \end{aligned}$$

This k is an integer for any value of n ≥ 2. $(i.e)$ k =Integer.

Case:2

Let $H_{2r+1,4n}$ be the graph with each vertex of degree is $2n+1$, if r=2 and then 4n vertices and $10n$ edges.

Suppose $H_{2r+1,4n}$ has a V-super vertex magic total labeling, then by theorem 2.2,

$$k = \frac{(p+q)(p+q+1)}{p} - \frac{p+1}{2}$$

In graph, $p = 4n$ and $q = 10n$

$$\begin{aligned} k &= \frac{(4n+10n)(4n+10n+1)}{4n} - \frac{4n+1}{2} \\ k &= \frac{(14n)(14n+1)}{4n} - \frac{4n+1}{2} \\ k &= 47n+3 \end{aligned}$$

This k is an integer for any value of n ≥ 2. $(i.e)$ k =Integer.

Case:3

Let $H_{2r+1,4n}$ be the graph with each vertex of degree is $2n+1$, if r=3 and then 4n vertices and $14n$ edges.

Suppose $H_{2r+1,4n}$ has a V-super vertex magic total labeling, then by theorem 2.2,

$$k = \frac{(p+q)(p+q+1)}{p} - \frac{p+1}{2}$$

In graph, $p = 4n$ *and* $q = 14n$

$$k = \frac{(4n + 14n)(4n + 14n + 1)}{4n} - \frac{4n + 1}{2}$$
$$k = \frac{(18n)(18n + 1)}{4n} - \frac{4n + 1}{2}$$
$$k = 79n + 4$$

This k *is an integer for any value of n* ≥ 2. $(i.e)$ k *=Integer.*

Hence $H_{2r+1,4n}$ *has a V-super vertex magic total labeling.*□

Theorem 3.2. *The graph* $H_{4,2n+5}$ *has a V-super vertex magic total labeling if* $n \geq 1$.

Proof.

Let $H_{4,2n+5}$ *be a regular graph with each vertex of degree is* 4.
The vertices are $2n + 5$ *and edges are* $4n + 10$.

Number of edges = 2 $*$ *Number of vertices.*

Suppose $H_{4,2n+5}$ *has a V-super vertex magic total labeling, then by theorem 2.2,*

$$k = \frac{(p + q)(p + q + 1)}{p} - \frac{p + 1}{2}$$

In graph, $p = 2n + 5$ *and* $q = 2 * p = 4n + 10$

$$k = \frac{((2n + 5) + (4n + 10))((2n + 5) + (4n + 10) + 1)}{2n + 5} - \frac{2n + 5 + 1}{2}$$

$$k = \frac{(6n + 15)(6n + 16)}{2n + 5} - \frac{2n + 6}{2}$$

$$k = 17n + 45$$

This is an integer if $n \geq 1$.
Hence $H_{4,2n+5}$ *has a V-super vertex magic total labeling.*□

Theorem 3.3. *The parity graph* P_a *has a V-super vertex magic total labeling.*

Proof.

Let P_a *be a parity graph with* 8 *vertices and* 12 *edges.*
Suppose P_a *has a E-super vertex magic total labeling, then by theorem 2.2,*

$$k = \frac{(p + q)(p + q + 1)}{p} - \frac{p + 1}{2}$$

Here $p = 8$ *and* $q = 12$

$$k = \frac{(8 + 12)(8 + 12 + 1)}{8} - \frac{8 + 1}{2}$$

$$k = \frac{(20)(21)}{8} - \frac{9}{2}$$

$$k = \frac{420}{8} - \frac{9}{2}$$

Since $\frac{420}{8}$ and $\frac{9}{2}$ is not an integer.But the summation value is integer.
Then,

$$\begin{aligned} k &= 52.5 - 4.5 \\ k &= 48 \\ k &= \textit{Integer.} \end{aligned}$$

Hence P_a the graph is V-super vertex magic total labeling. □

Theorem 3.4. *If any graph, then sum of the E-super vertex magic constant of graph and number of edges in graph is exactly equal to V-super vertex magic constant of graph.*

Proof.

Let G be a any graph with p vertices and q edges.

Consider E-super vertex magic graph (k),

$$k = q + \frac{(p+1)}{2} + \frac{q(q+1)}{p}$$

and Number of edges in graph G is denoted by q.

Hence V-super vertex magic graph(k),

$$\begin{aligned} k &= q + \frac{(p+1)}{2} + \frac{q(q+1)}{p} + q \\ k &= 2q + \frac{(p+1)}{2} + \frac{q(q+1)}{p} \end{aligned}$$

Adding and subtracting $\frac{(p+1)}{2}$, then by theorem 2.2,

$$\begin{aligned} k &= 2q + \frac{(p+1)}{2} + \frac{q(q+1)}{p} + \frac{(p+1)}{2} - \frac{(p+1)}{2} \\ k &= \frac{(p+q)(p+q+1)}{p} - \frac{p+1}{2} \end{aligned}$$

hence,

$$q + \frac{(p+1)}{2} + \frac{q(q+1)}{p} + q = \frac{(p+q)(p+q+1)}{p} - \frac{p+1}{2}$$

E-super vertex magic graph (k)+Number of edges in graph G = V-super vertex magic graph(k).

Hence,sum of the E-super vertex magic constant of graph and number of edges in graph is exactly equal to V-super vertex magic constant of graph.

This completes the proof. □

Theorem 3.5. *If $H_{m,n}(m = 3, n = 4(j+1)$ if $j = 1, 2, ...)$ is a V-super vertex magic graph, then labeling of outer edges are $p+1, p+2, ..., 2p$ and inner edges are $2p+1, 2p+2, ..., (p+q)$.*

Proof.

If G is V-super vertex magic labeling the vertex labeling are $1, 2, ..., p$. The edges labeling of graph G is $p+1, p+2, ..., 2p, ..., p+q$. The edge labeling of $H_{m,n}$ has two partitions into inner and outer edges.Let $H_{m,4n}$ be a regular graph with m degree and $4n$ edges.For $n = 2$, suppose G is V-super vertex magic graph, then the labeling of vertices are $1, 2, 3, ..., p$.Therefore, the only possibility for labeling of edges are $p+1, p+2, ..., 2p, ..., p+q$. Each vertex has at least one outer and one inner edges.The inner edges are(number of vertex) $/2$ then the maximum edge values are $2p+1, 2p+2, ..., (p+q)$. The remaining edges are outer edges, then the minimum edge values are $p+1, p+2, ..., 2p$. Hence $H_{m,n}$ is V-super vertex labeling.□

4 Examples

Example:4.1

$H_{3,8}$ graph is a V - Super vertex magic graph with magic number $k = 48$.

The assigning of vertex labels and edge labels of $H_{3,8}$ is tabulated as follows:

f:$V_i \to V_j$		V_0	V_1	V_2	V_3	V_4	V_5	V_6	V_7	K
		1	3	6	4	2	8	7	5	
V_0	1	-	15	-	-	18	-	-	14	48
V_1	3	15	-	13	-	-	17	-	-	48
V_2	6	-	13	-	9	-	-	20	-	48
V_3	4	-	-	9	-	16	-	-	19	48
V_4	2	18	-	-	16	-	12	-	-	48
V_5	8	-	17	-	-	12	-	11	-	48
V_6	7	-	-	20	-	-	11	-	10	48
V_7	5	14	-	-	19	-	-	10	-	48

Example:4.2

The Graph $H_{4,7}$ is a V-Super vertex magic graph with magic constant $K = 62$.

The assigning of vertex labels and edge labels of $H_{4,7}$ is tabulated as follows:

f:$V_i \to V_j$		V_0	V_1	V_2	V_3	V_4	V_5	V_6	K
		7	4	1	5	2	6	3	
V_0	7	-	18	13	-	-	9	15	62
V_1	4	18	-	21	8	-	-	11	62
V_2	1	13	21	-	17	10	-	-	62
V_3	5	-	8	17	-	20	12	-	62
V_4	2	-	-	10	20	-	16	14	62
V_5	6	9	-	-	12	16	-	19	62
V_6	3	15	11	-	-	14	19	-	62

Example:4.3

The Parity graph is a V-Super vertex magic graph with magic constant $k = 48$.

The assigning of vertex labels and edge labels of the parity graph P_a *is tabulated as follows:*

f:$V_i \to V_j$		V_0	V_1	V_2	V_3	V_4	V_5	V_6	V_7	K
		2	3	5	7	6	1	4	8	
V_0	2	-	10	-	-	-	17	19	-	48
V_1	3	10	-	20	-	-	-	-	15	48
V_2	5	-	20	-	14	-	-	-	9	48
V_3	7	-	-	14	-	11	-	-	16	48
V_4	6	-	-	-	11	-	18	13	-	48
V_5	1	17	-	-	-	18	-	12	-	48
V_6	4	19	-	-	-	13	12	-	-	48
V_7	8	-	15	9	16	-	-	-	-	48

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