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A CLASSIFICATION OF PARA-KENMOTSU SPACE FORMS

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Abstract A para-Kenmotsu space form is a para-Kenmotsu manifold with constant $\phi-$ sectional curvature. In this study, we work on para-Kenmotsu space forms under some flatness conditions of generalized quasi-conformal curvature tensor which is a generalization of conformal, concircular, conharmonic, projective, $\mathcal{M}$-projective and pseudo-projective curvature tensors. We examine GQC-flat and $\xi-$GQC flat p-Kenmotsu space forms.

1 Introduction

The contact geometry turns out to be a very fruitful branch of differential geometry, with many applications in different areas. After the tensorial approach used to understand the Riemannian geometry of contact manifolds, especially in the 1970s, there were many developments on the subject. Some different classes of contact manifolds were defined except for Sasakian manifolds. One of them was born in 1972 in a paper by Katsuei Kenmotsu, who proposed to study the properties of the warped product of the complex space with the real line. At that time the problem is natural since this product is one of the three classes in S. Tanno's [1] classification of connected almost contact Riemannian manifolds with automorphism group of maximum dimension. The manifolds were defined by Kenmotsu have been called Kenmotsu manifolds. Pitiş [2] wrote a comprehensive book in which articles on the subject were compiled.

A para-complex manifold M is a $2k-$dimensional differentiable manifold which has para-complex structure $J : TM \to TM, J^2 = I$. Similar to para-complex manifold the definition of an almost para-contact manifold was given by Sato [3]. As in the contact geometry, in the para-contact geometry, there are some subclasses of para-contact manifolds such as para-Sasakian manifolds [4], para-Kenmotsu manifolds [5]. A para-Kenmotsu manifold (briefly p-Kenmotsu) is a $(2k + 1)-$dimensional differentiable manifold that has a para-contact structure which is normal and satisfies Kenmotsu condition. p-Kenmotsu manifolds were defined and firstly studied by Sinha and Sai Prasad [5]. There many researches on the p-Kenmotsu manifolds as like [6, 7, 8, 9, 10, 11, 12].

The classification of geometric objects is one of the main purposes of geometry. In the Riemannian geometry, we can classify the manifolds by using the Riemann curvature. However, it is not always possible to find an exact expression of the Riemann curvature. Therefore, manifolds with non-zero curvature are interesting for Riemannian geometry. Riemann curvature tensor determines the sectional curvature of the manifold and the constancy of sectional curvature is important to understand the geometry of Riemannian manifolds. A Riemannian manifold of constant sectional curvature is called a space form. In the contact geometry, we have $\phi-$sectional curvature which is defined as the sectional curvature of the plane section $\{X, \phi X\}$. We recall a p-Kenmotsu manifold as a p-Kenmotsu space form if it has constant $\phi-$sectional curvature. In 1996, Dube [9] proved that a p-Kenmotsu space form is not flat and the Ricci tensor of such manifold is not parallel.

The main purpose of the presented paper is to classify p-Kenmotsu space forms under some flatness conditions via generalized quasi-conformal curvature (GQC) tensor. We examine GQC-flat and $\xi-$GQC flat p-Kenmotsu space forms. Also, we show that a $\xi-$concircularly flat p-Kenmotsu space form is locally isometric to the hyperbolic space.

2 PRELIMINARIES

In this section, we give some basic facts on p-Kenmotsu manifolds.

Definition 1. Let M be a $(2k+1)-$dimensional smooth manifold. (ϕ, ξ, η) is called an almost para-contact structure on M such that for ϕ is $(1,1)$ a tensor field, ξ is a vector field and η is a $1-$ form M we have

$$\phi^2 X = X - \eta(X)\xi, \ \phi(\xi) = 0, \ \eta \circ \phi = 0, \ \eta(\xi) = 1$$

where X is an arbitrary vector fields on M [2].

The tensor field ϕ induces an almost paracomplex structure on the distribution $D = ker\eta$. There are two distributions corresponding to the eigenvalues ± 1 and they are in same dimension k. The rank of ϕ is $2k$. A manifold with an almost para-contact structure is called an almost para-contact manifold [14].

Let g be a pseudo-Riemann metric on M.

- g is called compatible metric, if $g(\phi X_1, \phi X_2) = -g(X_1, X_2) + \eta(X_1)\eta(X_2)$ for all $X_1, X_2 \in \Gamma(TM)$
- g is called associated metric, if $d\eta(X_1, X_2) = g(\phi X_1, X_2)$ for all $X_1, X_2 \in \Gamma(TM)$.

If g is an associated metric then (M, ϕ, ξ, η, g) is called an almost para-contact metric manifold. Also by taking $X_2 = \xi$ we get $\eta(X_1) = g(X_1, \xi)$. The $2-$form $\Phi(X_1, X_2) = d\eta(X_1, X_2)$ is called second fundamental form. For a C^∞ function f on a para-contact metric manifold M, a complex structure J is defined on $M \times \mathbb{R}$ by $J(X_1, f\frac{d}{dt}) = (\phi X_1 + f\xi, \eta(X_1)\xi)$. If J is integrable i.e Nijenhuis tensor vanishes, then almost para-contact structure (ϕ, ξ, η) is called normal. Like contact manifolds this condition is restricted to $N_\phi + 2d\eta \otimes \xi = 0$.

A p-Kenmotsu manifold is an analogy of a Kenmotsu manifold [13] in para-contact geometry. These type of manifolds were defined by [5] as follow;

Definition 2. Let (M, ϕ, ξ, η, g) be an almost para-contact metric manifold of dimension $(2k+1)$. M is said to be an almost p-Kenmotsu manifold if $1-$form η is closed and $d\Phi = 2\eta \wedge \Phi$. A normal almost p-Kenmotsu manifold M is called a p-Kenmotsu manifold [5].

Theorem 2.1. *Let (M, ϕ, ξ, η, g) be an almost para-contact metric manifold. M is a p-Kenmotsu manifold if and only if*

$$(\nabla_{X_1}\phi) X_2 = g(\phi X_1, X_2)\xi - \eta(X_2)\phi X_1 \tag{2.1}$$

for all $X_1, X_2 \in \Gamma(TM)$ [15].

By using (2.1) on a p-Kenmotsu manifold we have

$$\nabla_{X_1}\xi = \phi^2 X_1$$

for all $X_1 \in \Gamma(TM)$.

A p-Kenmotsu manifold is called as a p-Kenmotsu space form if it has constant ϕ-sectional curvature c. A p-Kenmotsu space form has the following curvature expression [9];

$$\begin{aligned} R(X_1, X_2, X_3, X_4) \quad &= (\frac{c-3}{4})[g(X_2, X_3)g(X_1, X_4) - g(X_1, X_3)g(X_2, X_4)] \\ &+(\frac{c+1}{4})[g(X_2, \phi X_3)g(\phi X_1, X_4) - g(X_1, \phi X_3)g(\phi X_2, X_4) \\ &+2g(X_1, \phi X_2)g(\phi X_3, X_4) + \eta(X_1)\eta(X_3)g(X_2, X_4) - \eta(X_2)\eta(X_3)g(X_1, X_4) \\ &+\eta(X_2)\eta(X_4)g(X_1, X_3) - \eta(X_1)\eta(X_4)g(X_2, X_3)]. \end{aligned} \tag{2.2}$$

The Ricci and scalar curvatures of a p-Kenmotsu space form are given by

$$Ric(X_1, X_2) = (\frac{(k+1)c - 3k + 1}{4})g(X_1, X_2) - \frac{(k+1)(c+1)}{4}\eta(X_1)\eta(X_2) \tag{2.3}$$

and

$$scal = k(k+1)c - 3k^2 - k. \tag{2.4}$$

As we seen a p-Kenmotsu space form is an $\eta-$Einstein manifold.

Let take $X_2 = X_4 = \xi$ in (2.2). Thus, we obtain

$$R(X_1, \xi, X_3, \xi) = g(X_1, X_3) - \eta(X_1)\eta(X_3)$$

which can not equal to zero. Thus, we state

Theorem 2.2. *A p-Kenmotsu space form can not be flat [9].*

3 Generalized quasi-conformal flat p-Kenmotsu space forms

A Euclidean space is a manifold with zero Riemannian curvature tensor. This means that the Euclidean space is flat. Flatness is measured with the being zero of the Riemannian curvature tensor of a Riemannian manifold. If a Riemannian manifold is flat it is understood that the manifold is locally Euclidean. Therefore, flatness is an important notion for the classification of Riemannian manifolds. Some special maps can transform a Riemannian manifold into a Euclidean space. One of them is conformal maps. If a Riemannian manifold could be transformed to a Euclidean space with conformal maps then we recall this manifold by conformally flat. A conformal map has a curvature invariant which is called a conformal curvature tensor. Thus, we determine the conformally flatness with this tensor. Similar to the conformal curvature tensor we have different types of curvature tensors such as conformal, concircular, conharmonic, projective, m-projective, and pseudo-projective curvature tensors. In [18], a new curvature tensor which is called the generalized quasi-conformal (GQC) curvature tensor was defined. The GQC tensor on a p-Kenmotsu manifoldis given by

$$\begin{aligned}\tilde{C}(X_1, X_2)X_3 = R(X_1, X_2)X_3 + a[Ric(X_2, X_3)X_1 - Ric(X_1, X_3)X_2] \\ + b[g(X_2, X_3)QX_1 - g(X_1, X_3)QX_2] \\ - d\frac{scal}{2k+1}\left(\frac{1}{2k} + a + b\right)[g(X_2, X_3)X_1 - g(X_1, X_3)X_2]\end{aligned}$$

for all $X_1, X_2, X_3 \in \Gamma(TM)$, where R is the Riemannian curvature, Ric is Ricci curvature tensor, Q is the Ricci tensor, $scal$ is the scalar curvature and a, b, d are constants.

With the special values of a,b and d we get following special curvature tensors:

a	**b**	**d**	**curvature tensor**
0	0	0	Riemann curvature tensor (R)
$-\frac{1}{2k-1}$	$-\frac{1}{2k-1}$	1	Conformal curvature tensor (W)
$-\frac{1}{2k-1}$	$-\frac{1}{2k-1}$	0	Conharmonic curvature tensor $(\tilde{W})$
0	0	1	Concircular curvature tensor (E)
$-\frac{1}{2k}$	0	0	Projective curvature tensor (P)
$-\frac{1}{4k}$	$-\frac{1}{4k}$	0	$\mathcal{M}$-projective curvature tensor (H)

Riemannin manifold is said to be GQC flat if $\tilde{C} = 0$. The flatness of GQC curvature tensor also determines the flatness of special curvature tensors mentioned above table. GQC curvature tensor on some special manifolds have been studied in [16, 17, 18, 19].

By using curvature properties of a p-Kenmotsu space form we obtain the generalized quasi-conformal curvature tensor as

$$\begin{aligned}\tilde{C}(X_1, X_2, X_3, X_4) &= K_1\mathcal{T}_1(X_1, X_2, X_3, X_4) + K_2\mathcal{T}_2(X_1, X_2, X_3, X_4) \\ &\quad + K_3\mathcal{T}_3(X_1, X_2, X_3, X_4) + K_4\mathcal{T}_4(X_1, X_2, X_3, X_4)\end{aligned} \tag{3.1}$$

where $\mathcal{T}_1, \mathcal{T}_2, \mathcal{T}_3, \mathcal{T}_4$ are given by

$$\begin{aligned}
\mathcal{T}_1(X_1,X_2,X_3,X_4) &= g(X_2,X_3)g(X_1,X_4) - g(X_1,X_3)g(X_2,X_4)\\
\mathcal{T}_2(X_1,X_2,X_3,X_4) &= \eta(X_1)\eta(X_3)g(X_2,X_4) - \eta(X_2)\eta(X_3)g(X_1,X_4)\\
\mathcal{T}_3(X_1,X_2,X_3,X_4) &= \eta(X_2)\eta(X_4)g(X_1,X_3) - \eta(X_1)\eta(X_4)g(X_2,X_3)\\
\mathcal{T}_4(X_1,X_2,X_3,X_4) &= g(X_1,\phi X_3)g(\phi X_2,X_4) - g(X_1,\phi X_3)g(\phi X_2,X_4)\\
&\quad +2g(X_1,\phi X_2)g(\phi X_3,X_4)
\end{aligned}$$

and the coefficients K_1, K_2, K_3, K_4 are given by

$$\begin{aligned}
K_1 &= \frac{1}{4(2k+1)}\{[(2k+1-4(k+1)d+(k+1)(2k+1-2kd)(a+b))]c\\
&\quad +2[(2k+1)(-3k+1)+(3k^2+k)d](a+b)-3(2k+1)+4(k+1)d\}\\
K_2 &= (\frac{1-2(k+1)a}{4})(c+1), \;\; K_3 = (\frac{1-2(k+1)b}{4})(c+1), \;\; K_4 = \frac{c+1}{4}.
\end{aligned}$$

By using the special values of a, b and d we obtain the expression of conformal curvature tensor on a p-Kenmotsu manifold as follow;

$$\begin{aligned}
W(X_1,X_2,X_3,X_4) &= \frac{(-4k^2-6k+1)c+12k^2-5}{4(2k-1)(2k+1)}\mathcal{T}_1(X_1,X_2,X_3,X_4)\\
&\quad +(\frac{c+1}{4})\left[\frac{6k+3}{2k-1}(\mathcal{T}_2(X_1,X_2,X_3,X_4)\right.\\
&\quad \left.+\mathcal{T}_3(X_1,X_2,X_3,X_4))+\mathcal{T}_4(X_1,X_2,X_3,X_4)\right]
\end{aligned}$$

Similarly, the conharmonic curvature tensor is given by:

$$\begin{aligned}
\tilde{W}(X_1,X_2,X_3,X_4) &= \frac{(4k+1)c+6k-2}{4(2k-1)}\mathcal{T}_1(X_1,X_2,X_3,X_4)\\
&\quad +(\frac{c+1}{4})\left[\frac{6k+3}{2k-1}(\mathcal{T}_2(X_1,X_2,X_3,X_4)\right.\\
&\quad \left.+\mathcal{T}_3(X_1,X_2,X_3,X_4))+\mathcal{T}_4(X_1,X_2,X_3,X_4)\right]
\end{aligned}$$

The concircular curvature tensor is stated as:

$$\begin{aligned}
E(X_1,X_2,X_3,X_4) &= -\frac{2kc+2k-1}{4(2k-1)}\mathcal{T}_1(X_1,X_2,X_3,X_4)\\
&\quad +(\frac{c+1}{4})\left[\mathcal{T}_2(X_1,X_2,X_3,X_4)\right.\\
&\quad \left.+\mathcal{T}_3(X_1,X_2,X_3,X_4)+\mathcal{T}_4(X_1,X_2,X_3,X_4)\right]
\end{aligned}$$

The projective curvature tensor is obtained by:

$$\begin{aligned}
P(X_1,X_2,X_3,X_4) &= \frac{(k-1)c-2}{4}\mathcal{T}_1(X_1,X_2,X_3,X_4)\\
&\quad +(\frac{c+1}{4}(2k+1))\mathcal{T}_2(X_1,X_2,X_3,X_4)\\
&\quad +\frac{c+1}{4}\left[\mathcal{T}_3(X_1,X_2,X_3,X_4)+\mathcal{T}_4(X_1,X_2,X_3,X_4)\right].
\end{aligned}$$

Finally, the $\mathcal{M}$-projective curvature tensor is given by:

$$\begin{aligned}
H(X_1,X_2,X_3,X_4) &= \frac{(k-1)c-12k^2-2}{8k}\mathcal{T}_1(X_1,X_2,X_3,X_4)\\
&\quad +(\frac{c+1}{4}(2k+1))\mathcal{T}_2(X_1,X_2,X_3,X_4)\\
&\quad +\frac{c+1}{4}\left[\mathcal{T}_3(X_1,X_2,X_3,X_4)+\mathcal{T}_4(X_1,X_2,X_3,X_4)\right].
\end{aligned}$$

for all $X_1, X_2, X_3, X_4 \in \Gamma(TM)$.

Suppose that generalized quasi-conformal curvature tensor vanishes on a p-Kenmotsu space form M. Then from (3.1), we get

$$\begin{aligned} 0 &= K_1\mathcal{T}_1(X_1, X_2, X_3, X_4) + K_2\mathcal{T}_2(X_1, X_2, X_3, X_4) \\ &\quad + K_3\mathcal{T}_3(X_1, X_2, X_3, X_4) + K_4\mathcal{T}_4(X_1, X_2, X_3, X_4). \end{aligned} \tag{3.2}$$

Let take an orthonormal basis of $\Gamma(TM)$ as $\{E_1, ..., E_k, E_{k+1} = \phi E_1, ..., E_{2k} = \phi E_k, E_{2k+1} = \xi\}$. By setting $X_2 = X_3 = E_i$ and getting sum over i such that $1 \leq i \leq 2k+1$ in (3.2), we get

$$(2kK_1 - K_2 + 3K_4)g(X_1, X_4) + (K_2 - 2kK_3 - 3K_4)\eta(X_1)\eta(X_4) = 0.$$

Choosing $X_1 = X_4 = \xi$ we obtain $K_1 - K_3 = 0$. Thus, we state

Theorem 3.1. *On a GQC-flat p-Kenmotsu space form we have* $K_1 - K_3 = 0$.

Let take $\mathcal{W} = 0$. Then with solving the equation $K_1 - K_3 = 0$ we get $c = -\frac{2-4k^2}{3k}$. Similarly,

(i) if $\tilde{W} = 0$, then we get $c = \frac{1-4k}{12k+3}$.

(ii) if $E = 0$, then we get $c = \frac{-4k}{4k+1}$.

(iii) if $P = 0$, then we get $c = \frac{3}{k}$.

(iv) if $H = 0$, then we get $c = \frac{-12k^2+2k+2}{k+1}$.

Therefore, we provide the following corollary;

Corollary 1. Let M be a p-Kenmotsu space form. If M is

- conformally flat then $c = -\frac{2-4k^2}{3k}$,
- conharmonically-flat then $c = \frac{1-4k}{12k+3}$,
- concircularly-flat then $c = \frac{-4k}{4k+1}$,
- projectively-flat then $c = \frac{3}{k}$,
- $\mathcal{M}-$projectively-flat then $c = \frac{-12k^2+2k+2}{k+1}$.

Remark 3.2. If we take $a = b = d = 0$, then we obtain $K_1 - K_3 = \frac{c}{4} - 6k - 3 - \frac{c+1}{4} = 0$. This is provided that $k = -\frac{13}{8}$ which is not possible. This gives that M do not have zero Riemannian curvature tensor which is compatible with Theorem 2.2.

4 $\xi-$generalized quasi-conformal flat p-Kenmotsu space forms

Let us consider the generalized quasi-conformal curvature tensor a map such as

$$\tilde{C} : T_pM \times T_pM \times T_pM \to \phi(T_pM) \oplus \{\xi\}$$

Three particular cases can be considered as follows:

(i) $\tilde{C} : T_pM \times T_pM \times T_pM \to \{\xi\}$ that is, the projection of the image of $\tilde{C}$ in $\phi(T_pM)$ is zero.

(ii) $\tilde{C} : T_pM \times T_pM \times T_pM \to \phi(T_pM)$ that is, the projection of the image of $\tilde{C}$ in $\{\xi\}$ is zero.

(iii) $\tilde{C} : \phi(T_pM) \times \phi(T_pM) \times \phi(T_pM) \to \{\xi\}$ that is, when $\tilde{C}$ is restricted to $\phi(\phi(T_pM) \times \phi(T_pM))$, the projection of the image of $\tilde{C}$ in $\phi(T_pM)$ is zero, which is equivalent to $\phi^2\tilde{C}(\phi X_1, \phi X_2)\phi X_3 = 0$.

On a para-contact metric manifold we have $\eta(\phi(TM)) = d\eta(\xi, TM) = 0$. Conversely, if $\eta(X) = 0$, then $X = -\phi^2 X \in \phi(TM)$. The generalized quasi-conformal curvature tensor with respect to the metric g is the tensor field of type $(1,3)$. On the other hand, the Lie algebra TM can be decomposed in a direct sum $TM = \phi(TM) \oplus \{\xi\}$, where $\{\xi\}$ is the 1-dimensional distribution on M generated by the characteristic field ξ. In [20], the authors defined $\xi-$conformally flatness. Similarly, we can define $\xi-$generalized quasi-conformal flatness as follow;

Definition 4.1. A para-contact metric manifold $(M^{2k+1}, \phi, \xi, \eta, g)$ is said to be $\xi-$ conformally flat if the linear operator $\tilde{C}(X_1, X_2)$ is an endomorphism of $\phi(TM)$, that is, if $\tilde{C}(X_1, X_2)\phi(TM) \subset \phi(TM)$ [20].

In [20], it was proved that $\xi-$conformally flatness of an almost contact manifold is equivalet to the condition $W(X_1, X_2)\xi = 0$. Thus we call p-Kenmotsu space form as $\xi - \tilde{C}$-flat if $\tilde{C}(X_1, X_2)\xi = 0$.

Theorem 4.2. *A p-Kenmotsu space form is $\xi - \tilde{C}$-flat if and only if $K_1 - K_2 = 0$.*

Proof. Suppose that $\tilde{C}(X_1, X_2)\xi = 0$, then we obtain

$$\begin{aligned} \tilde{C}(X_1, X_2, \xi, X_4) &= K_1\mathcal{T}_1(X_1, X_2, \xi, X_4) + K_2\mathcal{T}_2(X_1, X_2, \xi, X_4) \\ &\quad + K_3\mathcal{T}_3(X_1, X_2, \xi, X_4) + K_4\mathcal{T}_4(X_1, X_2, \xi, X_4) = 0 \end{aligned}$$

for all $X_1, X_2, X_4 \in \Gamma(TM)$. Thus, since $\mathcal{T}_3(X_1, X_2, \xi, X_4) = \mathcal{T}_4(X_1, X_2, \xi, X_4) = 0$ we get

$$(K_1 - K_2)(\eta(X_2)g(X_1, X_4) - \eta(X_1)g(X_2, X_4)) = 0. \tag{4.1}$$

Let take an orthonormal frame of $\Gamma(TM)$ as $\{E_1, E_2, ..., E_k, E_{k+1} = \phi E_1, ..., E_{2k} = \phi E_k, \xi\}$. By taking $X_1 = X_4 = E_i$ and getting sum of (4.1) over i, $1 \leq i \leq 2k + 1$ we have

$$0 = 2k(K_1 - K_2)\eta(X_2).$$

Finally we get $K_1 - K_2 = 0$. □

Let take $W(X_1, X_2)\xi = 0$, then after solving the equation $K_1 - K_2 = 0$ we get $c = -\frac{3k+4}{8k^2+9k+1}$. Similarly,

(i) if $\tilde{W}(X_1, X_2)\xi = 0$, then we get $c = \frac{-18k+7}{10k+1}$,

(ii) if $P(X_1, X_2)\xi = 0$, then we get $c = \frac{2k+3}{k+2}$,

(iii) if $H(X_1, X_2)\xi = 0$, then we get $c = \frac{12k^2-4k}{5k+1}$.

Therefore, we provide the following corollary;

Corollary 2. Let M be a p-Kenmotsu space form. Then if M is

- $\xi-$conformally flat then $c = -\frac{3k+4}{8k^2+9k+1}$,
- $\xi-$conharmonically-flat then $c = \frac{-18k+7}{10k+1}$,
- $\xi-$projectively-flat then $c = \frac{2k+3}{k+2}$,
- $\xi-\mathcal{M}-$projectively-flat then $c = \frac{12k^2-4k}{5k+1}$.

On the other hand if $E(X_1, X_2)\xi = 0$, then we get $c = -1$. Thus, from (2.2) we have,

$$R(X_1, X_2, X_3, X_4) = -[g(X_2, X_3)g(X_1, X_4) - g(X_1, X_3)g(X_2, X_4)].$$

Finally, we conclude that

Theorem 4.3. *$\xi-$concircularly-flat p-Kenmotsu space form is locally isometric to hyperbolic space $\mathbb{H}^{2k+1}(-1)$.*

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