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A STUDY ON PACKING COLORING OF FAN AND JUMP GRAPH FAMILIES

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Abstract This paper deals with the packing chromatic number of the middle, total, central and line graph of the fan graph and the jump graph of the following graphs: path, cycle, wheel, double wheel, helm, closed helm and sunlet graph.

1 Introduction

Graph theory is one of the most flourishing branches of mathematics with applications to a wide variety of subjects. An assignment of colors to the vertices of a graph so that no two adjacent vertices get the same color is called a coloring of the graph. Usually a given graph can be colored in many different ways. One such way is packing coloring.

The area of frequency assignment in wireless networks issues the concept of packing coloring which was introduced by Wayne Goddard et al.[4] in 2007 under the name broadcast coloring. It has several operations like resource placements and biological diversity. Bresar et al.[1] introduced the term packing chromatic number χ_ρ.

A packing k- coloring of a graph G[1,2] is a mappingπ from V(G) to $\{1, 2, ..., k\}$ such that any two vertices of color i are at distance at least $i + 1$. The packing chromatic number χ_ρ of a graph G is the smallest integer k for which G has packing k- coloring. In a particular network the signals that are using the identical broadcast frequency of two different stations will hinder unless they are situated adequacy far apart. The distance in which the signals will broadcast is directly associated to the power of those signals. Bresar et al. [1] stated that this concept might have numerous added applications-for example, in resource placement and biological diversity (diverse species in a particular area need diverse amounts of territory).

The packing coloring problem is NP-complete for general graphs which is justified by Goddard et al.[4] and it is NP-complete even for trees is confirmed by Fiala and Golovach [3].

The line graph [5] of G denoted by L(G) is the intersection graph of the edges of G, representing each edge by the set of its two end vertices. Otherwise L(G) is a graph such that

(i) Each vertex of L(G) represents an edge of G.

(ii) Two vertices of L(G) are adjacent if their corresponding edges share a common end point in G.

The middle graph [7] of G, denoted by M(G) is the graph whose vertex set is $V(G) \cup E(G)$ where two vertices are adjacent if

(i) They are adjacent edges of G or

(ii) One is a vertex of G and the other is an edge incident with it.

The total graph [7,11] of G, is the graph whose vertex set is $V(G) \cup E(G)$ and two vertices are adjacent whenever they are either adjacent or incident in G.

The central graph [11] of a graph G denoted by C(G) is formed by adding an extra vertex on each edge of G, and joining each pair of vertices of the original graph which were previously non-adjacent.

The jump graph [6] J(G) of G is the graph whose vertices are edges of G, and where two vertices of J(G) are adjacent if and only if they are not adjacent in G. Equivalently, the Jump graph J(G) of G is the complement of line graph of G.

A fan graph $F_{n,m}$ [12] is defined as the graph join $\bar{K_n} + P_m$, where $\bar{K_n}$ is the empty graph on n vertices and P_m is the path graph on m vertices. In particular, when n=1 the graph $F_{1,m}$ is called the fan graph of order m.

Rajalakshmi and Venkatachalam [8,9,10] have discussed the concept of packing coloring of double wheel graph families. Further, this paper exhibits the packing coloring of Fan and Jump graph of certain graphs.

2 Packing coloring of middle, total, central and line graph of Fan graph

Theorem 2.1. *If $m \geq 5$, then the packing chromatic number of the middle graph of fan graph is*
$$\chi_\rho[M(F_{1,m})] = \begin{cases} \frac{5m+3}{3} & \text{if } m \equiv 0 \bmod 3 \\ \frac{5m+4}{3} & \text{if } m \equiv 1 \bmod 3 \\ \frac{5m+2}{3} & \text{if } m \equiv 2 \bmod 3 \end{cases}$$

Proof. Let us assume that $V(F_{1,m}) = \{v, v_l : 1 \leq l \leq m\}$ and $E(F_{1,m}) = \{w_l : 1 \leq l \leq m\} \cup \{x_l : 1 \leq l \leq m-1\}$, where w_l is the edge corresponding to vv_l and x_l is the edge corresponding to v_lv_{l+1}. $V[M(F_{1,m})] = \{v, v_l, u_l : 1 \leq l \leq m\} \cup \{u_l^{'} : 1 \leq l \leq m-1\}$
For $1 \leq l \leq m$

- Each edge vv_l is subdivided by u_l of $F_{1,m}$
- Each edge v_lv_{l+1} is subdivided by $u_l^{'}$ of $F_{1,m}$

Let us prove the lower bound of $\chi_\rho[M(F_{1,m})]$ by contradiction method. In $\chi_\rho[M(F_{1,m})]$, we have to prove three cases. So, we assume $\chi_\rho[M(F_{1,m})] < \frac{5m+3}{3}$, $\frac{5m+4}{3}$ and $\frac{5m+2}{3}$ for $m \equiv 0 \bmod 3$, $m \equiv 1 \bmod 3$ and $m \equiv 2 \bmod 3$ respectively. Now, we need to select the colors for each valid vertex in all cases of $\chi_\rho[M(F_{1,m})]$ as $\frac{5m}{3}$, $\frac{5m+1}{3}$ and $\frac{5m-1}{3}$ for $m \equiv 0 \bmod 3$, $m \equiv 1 \bmod 3$ and $m \equiv 2 \bmod 3$ respectively. From middle graph of fan graph, we observed $d(u_l, u_l^{'}) = 1, d(v_l, u_{l+1}) = 2$ and $d(u_{3l-2}^{'}, u_{3l+1}^{'}) = 3$ are true in all the cases. Here the maximum distance is 3 and the diameter of $\chi_\rho[M(F_{1,m})]$ is also 3. This shows that we can repeat only the colors 1 and 2 in $\chi_\rho[M(F_{1,m})]$. Since, we assumed $\frac{5m}{3}$, $\frac{5m+1}{3}$ and $\frac{5m-1}{3}$ for $m \equiv 0 \bmod 3$, $m \equiv 1 \bmod 3$ and $m \equiv 2 \bmod 3$ respectively. Therefore, we are left with remaining colors as $\frac{5m-6}{3}$, $\frac{5m-5}{3}$ and $\frac{5m-7}{3}$ for $m \equiv 0 \bmod 3$, $m \equiv 1 \bmod 3$ and $m \equiv 2 \bmod 3$ respectively. According to the definition of packing coloring, if two vertices of color i are at distance of at least $i+1$ apart and $d(v_l, u_{l+1}) = 2$ then $c(v_l) \neq c(u_{l+1})$ and the following colors are required $\frac{5m-6}{3}$, $\frac{5m-5}{3}$ and $\frac{5m-7}{3}$ for $m \equiv 0 \bmod 3$, $m \equiv 1 \bmod 3$ and $m \equiv 2 \bmod 3$ respectively for each v_l and u_{l+1}. While this proves a contradictory value compared to the desired output, the statement $\chi_\rho[M(F_{1,m})] < \frac{5m+3}{3}$, $\frac{5m+4}{3}$ and $\frac{5m+2}{3}$ for $m \equiv 0 \bmod 3$, $m \equiv 1 \bmod 3$ and $m \equiv 2 \bmod 3$ respectively are wrong. Hence, we conclude that the lower bound
$$\chi_\rho[M(F_{1,m})] \geq \begin{cases} \frac{5m+3}{3} \; if \; m \equiv 0 \bmod 3 \\ \frac{5m+4}{3} \; if \; m \equiv 1 \bmod 3 \\ \frac{5m+2}{3} \; if \; m \equiv 2 \bmod 3 \end{cases}$$
Next, we need to calculate the upper bound
$$\chi_\rho[M(F_{1,m})] \leq \begin{cases} \frac{5m+3}{3} \; if \; m \equiv 0 \bmod 3 \\ \frac{5m+4}{3} \; if \; m \equiv 1 \bmod 3 \\ \frac{5m+2}{3} \; if \; m \equiv 2 \bmod 3 \end{cases}$$
Consider the color function,
$c : V(M(F_{1,m})) \rightarrow \{1, 2, 3, ..., p\}$ is defined as follows,
$$\text{where } p = \begin{cases} \frac{5m+3}{3} \; if \; m \equiv 0 \bmod 3 \\ \frac{5m+4}{3} \; if \; m \equiv 1 \bmod 3 \\ \frac{5m+2}{3} \; if \; m \equiv 2 \bmod 3 \end{cases}$$

$$\begin{array}{ll} c(v) = c(v_l) = 1 & \text{for } 1 \leq l \leq m \\ c(u_{3l-2}^{'}) = 2 & \text{for } 1 \leq l \leq m \\ c(u_{3l-1}^{'}) = 2l+1 & \text{for } 1 \leq l \leq m \\ c(u_{3l}^{'}) = 2l+2 & \text{for } 1 \leq l \leq m \end{array}$$

If $m \equiv 0 \; mod \; 3$

$$c(u_l) = \frac{m+3l+9}{3} \qquad \text{for } 1 \leq l \leq m$$

If $m \equiv 1 \; mod \; 3$

$$c(u_l) = \frac{m+3l+11}{3} \qquad \text{for } 1 \leq l \leq m$$

If $m \equiv 2 \; mod \; 3$

$$c(u_l) = \frac{m+3l+7}{3} \qquad \text{for } 1 \leq l \leq m$$

Therefore, the upper bound

$$\chi_\rho[M(F_{1,m})] \leq \begin{cases} \frac{5m+3}{3} \; if \; m \equiv 0 \; mod \; 3 \\ \frac{5m+4}{3} \; if \; m \equiv 1 \; mod \; 3 \\ \frac{5m+2}{3} \; if \; m \equiv 2 \; mod \; 3 \end{cases}$$

Hence, $\chi_\rho[M(F_{1,m})] = \begin{cases} \frac{5m+3}{3} \; if \; m \equiv 0 \; mod \; 3 \\ \frac{5m+4}{3} \; if \; m \equiv 1 \; mod \; 3 \\ \frac{5m+2}{3} \; if \; m \equiv 2 \; mod \; 3 \end{cases}$ □

Theorem 2.2. *If $m \geq 6$, then the packing chromatic number of the total graph of fan graph is*

$$\chi_\rho[T(F_{1,m})] = \begin{cases} \frac{13m+6}{6} & \text{if } m \equiv 0 \mod 6 \\ \frac{13m+5}{6} & \text{if } m \equiv 1 \mod 6 \\ \frac{13m+4}{6} & \text{if } m \equiv 2 \mod 6 \\ \frac{13m+3}{6} & \text{if } m \equiv 3 \mod 6 \\ \frac{13m+2}{6} & \text{if } m \equiv 4 \mod 6 \\ \frac{13m+1}{6} & \text{if } m \equiv 5 \mod 6 \end{cases}$$

Proof. Let us assume that $V(F_{1,m}) = \{v, v_l : 1 \leq l \leq m\}$ and $E(F_{1,m}) = \{w_l : 1 \leq l \leq m\} \cup \{x_l : 1 \leq l \leq m-1\}$, where w_l is the edge corresponding to vv_l and x_l is the edge corresponding to $v_l v_{l+1}$. $V[T(F_{1,m})] = \{v, v_l, u_l : 1 \leq l \leq m\} \cup \{u_l^{'} : 1 \leq l \leq m-1\}$.
For $1 \leq l \leq m$

- Each edge vv_l is subdivided by u_l of $F_{1,m}$
- Each edge $v_l v_{l+1}$ is subdivided by $u_l^{'}$ of $F_{1,m}$

Let us prove the lower bound of $\chi_\rho[T(F_{1,m})]$ by the contradiction method. In $\chi_\rho[T(F_{1,m})]$, we have to prove six cases. So, we assume $\chi_\rho[T(F_{1,m})] < \frac{13m+6}{6}, \frac{13m+5}{6}, \frac{13m+4}{6}, \frac{13m+3}{6}, \frac{13m+2}{6}, \frac{13m+1}{6}$ for $m \equiv 0 \; mod \; 6$, $m \equiv 1 \; mod \; 6$, $m \equiv 2 \; mod \; 6$, $m \equiv 3 \; mod \; 6$, $m \equiv 4 \; mod \; 6$, $m \equiv 5 \; mod \; 6$ respectively. Now, we need to select the colors for each valid vertex in $\chi_\rho[T(F_{1,m})]$ as $\frac{13m}{6}, \frac{13m-1}{6}, \frac{13m-2}{6}, \frac{13m-3}{6}, \frac{13m-4}{6}, \frac{13m-5}{6}$ for $m \equiv 0 \; mod \; 6$, $m \equiv 1 \; mod \; 6$, $m \equiv 2 \; mod \; 6$, $m \equiv 3 \; mod \; 6$, $m \equiv 4 \; mod \; 6$, $m \equiv 5 \; mod \; 6$ respectively. Bt the definition of total graph of fan graph, we observed that $c(v_{2l-1}) = 1$ and $d(u_l, u_l^{'}) = 1$, $d(v_{l+4}, u_{l+3}) = 2$ and $d(u_{3l-2}^{'}, u_{3l+1}^{'}) = 3$ are true in all the cases. Here the maximum distance is 3 and the diameter of $\chi_\rho[T(F_{1,m})]$ is also 3. This shows that we can repeat only the colors 1 and 2 in $\chi_\rho[T(F_{1,m})]$. Since, we assumed $\frac{13m}{6}, \frac{13m-1}{6}, \frac{13m-2}{6}, \frac{13m-3}{6}, \frac{13m-4}{6}, \frac{13m-5}{6}$ for $m \equiv 0 \; mod \; 6, m \equiv 1 \; mod \; 6, m \equiv 2 \; mod \; 6, m \equiv 3 \; mod \; 6, m \equiv 4 \; mod \; 6, m \equiv 5 \; mod \; 6$ respectively. Therefore, we are left with a remaining colors as $\frac{13m-12}{6}, \frac{13m-13}{6}, \frac{13m-14}{6}, \frac{13m-15}{6}, \frac{13m-16}{6}, \frac{13m-17}{6}$ for $m \equiv 0 \; mod \; 6, m \equiv 1 \; mod \; 6, m \equiv 2 \; mod \; 6, m \equiv 3 \; mod \; 6, m \equiv 4 \; mod \; 6, m \equiv 5 \; mod \; 6$ respectively. According to the definition of packing coloring, if two vertices of color i are at distance of at least $i+1$ apart and $d(v_{l+4}, u_{l+3}) = 2$ then $c(v_{l+4}) \neq c(u_{l+3})$ and the following colors are required $\frac{13m-12}{6}, \frac{13m-13}{6}, \frac{13m-14}{6}, \frac{13m-15}{6}, \frac{13m-16}{6}, \frac{13m-17}{6}$ for $m \equiv 0 \; mod \; 6, m \equiv 1 \; mod \; 6, m \equiv 2 \; mod \; 6, m \equiv 3 \; mod \; 6, m \equiv 4 \; mod \; 6, m \equiv 5 \; mod \; 6$ respectively for each v_{l+4} and u_{l+3}. While this proves a contradictory value compared to the desired output, the statement $\chi_\rho[T(F_{1,m})] < \frac{13m+6}{6}, \frac{13m+5}{6}, \frac{13m+4}{6}, \frac{13m+3}{6}, \frac{13m+2}{6}, \frac{13m+1}{6}$ for $m \equiv 0 \; mod \; 6, m \equiv 1 \; mod \; 6, m \equiv 2 \; mod \; 6, m \equiv 3 \; mod \; 6, m \equiv 4 \; mod \; 6, m \equiv 5 \; mod \; 6$ respectively are wrong.

Hence, we conclude that the lower bound $\chi_\rho[T(F_{1,m})] \geq \begin{cases} \frac{13m+6}{6} \; if \; m \equiv 0 \; mod \; 6 \\ \frac{13m+5}{6} \; if \; m \equiv 1 \; mod \; 6 \\ \frac{13m+4}{6} \; if \; m \equiv 2 \; mod \; 6 \\ \frac{13m+3}{6} \; if \; m \equiv 3 \; mod \; 6 \\ \frac{13m+2}{6} \; if \; m \equiv 4 \; mod \; 6 \\ \frac{13m+1}{6} \; if \; m \equiv 5 \; mod \; 6 \end{cases}$

Next, we need to calculate the upper bound $\chi_\rho[T(F_{1,m})] \leq \begin{cases} \frac{13m+6}{6} \; if \; m \equiv 0 \; mod \; 6 \\ \frac{13m+5}{6} \; if \; m \equiv 1 \; mod \; 6 \\ \frac{13m+4}{6} \; if \; m \equiv 2 \; mod \; 6 \\ \frac{13m+3}{6} \; if \; m \equiv 3 \; mod \; 6 \\ \frac{13m+2}{6} \; if \; m \equiv 4 \; mod \; 6 \\ \frac{13m+1}{6} \; if \; m \equiv 5 \; mod \; 6. \end{cases}$

Consider the color function,
$c : V(T(F_{1,m})) \rightarrow \{1, 2, 3, ..., p\}$ is defined as follows,
where $p = \begin{cases} \frac{13m+6}{6} \; if \; m \equiv 0 \; mod \; 6 \\ \frac{13m+5}{6} \; if \; m \equiv 1 \; mod \; 6 \\ \frac{13m+4}{6} \; if \; m \equiv 2 \; mod \; 6 \\ \frac{13m+3}{6} \; if \; m \equiv 3 \; mod \; 6 \\ \frac{13m+2}{6} \; if \; m \equiv 4 \; mod \; 6 \\ \frac{13m+1}{6} \; if \; m \equiv 5 \; mod \; 6 \end{cases}$

$$\begin{array}{ll} c(v_{2l-1}) = 1 & \text{for } 1 \leq l \leq m \\ c(v) = 3 & \\ c(v_n) = 2 & \text{for } m \equiv 1 \; mod \; 3 \\ c(u'_{3l-2}) = 2 & \text{for } 1 \leq l \leq m-1 \\ c(u'_{3l-1}) = 2l+2 & \text{for } 1 \leq l \leq m-1 \\ c(u'_{3l}) = 2l+3 & \text{for } 1 \leq l \leq m-1 \end{array}$$

For even vertices of v_l
i) If $m \equiv 0 \; mod \; 6$ and if $m \equiv 3 \; mod \; 6$

$$c(v_{2l}) = \frac{4m+6l+12}{6} \qquad \text{for } 1 \leq l \leq m$$

ii) If $m \equiv 1 \; mod \; 6$ and if $m \equiv 4 \; mod \; 6$

$$c(v_{2l}) = \frac{4m+6l+14}{6} \qquad \text{for } 1 \leq l \leq m$$

iii) If $m \equiv 2 \; mod \; 6$ and if $m \equiv 5 \; mod \; 6$

$$c(v_{2l}) = \frac{4m+6l+10}{6} \qquad \text{for } 1 \leq l \leq m$$

For the vertices of u_l
$c(u_2) = 1$
i) If $m \equiv 0 \; mod \; 6$ and if $m \equiv 1 \; mod \; 6$

$$\begin{array}{ll} c(u_l) = \frac{3m+6l+\lceil \frac{m}{2} \rceil+3}{3} & l = 1 \\ c(u_l) = \frac{3m+3l+\lceil \frac{m}{2} \rceil+3}{3} & \text{for } 3 \leq l \leq m \end{array}$$

ii) If $m \equiv 2 \; mod \; 6$ and if $m \equiv 3 \; mod \; 6$

$$\begin{array}{ll} c(u_l) = \frac{3m+6l+\lceil \frac{m}{2} \rceil+2}{3} & l = 1 \\ c(u_l) = \frac{3m+3l+\lceil \frac{m}{2} \rceil+2}{3} & \text{for } 3 \leq l \leq m \end{array}$$

iii) If $m \equiv 4 \; mod \; 6$ and if $m \equiv 5 \; mod \; 6$

$$\begin{array}{ll} c(u_l) = \frac{3m+6l+\lceil \frac{m}{2} \rceil+1}{3} & l = 1 \\ c(u_l) = \frac{3m+3l+\lceil m2 \rceil+1}{3} & \text{for } 3 \leq l \leq m \end{array}$$

Therefore, the upper bound of

$$\chi_\rho[T(F_{1,m})] \leq \begin{cases} \frac{13m+6}{6} \; if \; m \equiv 0 \; mod \; 6 \\ \frac{13m+5}{6} \; if \; m \equiv 1 \; mod \; 6 \\ \frac{13m+4}{6} \; if \; m \equiv 2 \; mod \; 6 \\ \frac{13m+3}{6} \; if \; m \equiv 3 \; mod \; 6 \\ \frac{13m+2}{6} \; if \; m \equiv 4 \; mod \; 6 \\ \frac{13m+1}{6} \; if \; m \equiv 5 \; mod \; 6 \end{cases}$$

Hence, $\chi_\rho[T(F_{1,m})] = \begin{cases} \frac{13m+6}{6} \; if \; m \equiv 0 \; mod \; 6 \\ \frac{13m+5}{6} \; if \; m \equiv 1 \; mod \; 6 \\ \frac{13m+4}{6} \; if \; m \equiv 2 \; mod \; 6 \\ \frac{13m+3}{6} \; if \; m \equiv 3 \; mod \; 6 \\ \frac{13m+2}{6} \; if \; m \equiv 4 \; mod \; 6 \\ \frac{13m+1}{6} \; if \; m \equiv 5 \; mod \; 6 \end{cases}$ □

Theorem 2.3. *If $m \geq 3$, then the packing chromatic number of the central graph of fan graph is $\chi_\rho[C(F_{1,m})] = m + 2$.*

Proof. Let us assume that $V(F_{1,m}) = \{v, v_l : 1 \leq l \leq m\}$ and $E(F_{1,m}) = \{w_l : 1 \leq l \leq m\} \cup \{x_l : 1 \leq l \leq m-1\}$, where w_l is the edge corresponding to vv_l and x_l is the edge corresponding to $v_l v_{l+1}$. $V[C(F_{1,m})] = \{v, v_l, u_l, u_l^{'} : 1 \leq l \leq m\} \cup \{u_l^{'} : 1 \leq l \leq m-1\}$
For $1 \leq l \leq m$

- Each edge vv_l is subdivided by u_l of $F_{1,m}$
- Each edge $v_l v_{l+1}$ is subdivided by $u_l^{'}$ of $F_{1,m}$

Let us prove the lower bound by the contradiction method. In $\chi_\rho[C(F_{1,m})]$, we assume $\chi_\rho[C(F_{1,m})] < m + 2$ and we need to select $m + 1$ colors for each valid vertex in $C(F_{1,m})$. By the definition of central graph of fan graph, we have two rules $c(u_l) = c(u_l^{'}) = c_1, d(v_l, u_l^{'}) = d(v_{l+1}, u_l^{'}) = d(v_l, v_{l+1}) = d(v, u_l) = d(v_l, u_l) = 1$ and $(v_l, u_{l+2}) = 2$. Here the maximum distance is 2 and the diameter of $\chi_\rho[C(F_{1,m})]$ is also 2. This shows that we can repeat only the colors 1 in $\chi_\rho[C(F_{1,m})]$. After the selection of $m + 1$ colors we are left with a remaining of m colors. According to the definition of packing coloring, if two vertices of color i are at distance of at least $i + 1$ apart and $d(v_l, u_{l+2}) = 2$ then $c(v_l) \neq c(u_{l+2})$ and m colors are required for each v_l and v_{l+1}. While this proves a contradictory value compared to the desired output, the statement $\chi_\rho[C(F_{1,m})] < m + 2$ is wrong.
Hence, we conclude that the lower bound of $\chi_\rho[C(F_{1,m})] \geq m + 2$. Next, we need to calculate the upper bound $\chi_\rho[C(F_{1,m})] \leq m + 2$.
On considering the color function $c : V[C(F_{1,m})] \to \{1, 2, ..., m + 2\}$ is defined by,

$$\begin{array}{ll} c(u_l) = 1 & \text{for } 1 \leq l \leq m \\ c(u_l^{'}) = 1 & \text{for } 1 \leq l \leq m-1 \\ c(v) = 2 & \\ c(u_l) = l + 2 & \text{for } 1 \leq l \leq m \end{array}$$

Therefore, the upper bound $\chi_\rho[C(F_{1,m})] \leq m + 2$. Hence, $\chi_\rho[C(F_{1,m})] = m + 2$. □

Theorem 2.4. *If $m \geq 4$, then the packing chromatic number of the line graph of fan graph is*

$$\chi_\rho[L(F_{1,m})] = \begin{cases} \frac{5m+4}{4} & \text{if } m \equiv 0 \mod 4 \\ \frac{5m+3}{4} & \text{if } m \equiv 1 \mod 4 \\ \frac{5m+2}{4} & \text{if } m \equiv 2 \mod 4 \\ \frac{5m+1}{4} & \text{if } m \equiv 3 \mod 4 \end{cases}$$

Proof. Let us assume that $V(F_{1,m}) = \{v, v_l : 1 \leq l \leq m\}$, $E(F_{1,m}) = \{w_l : 1 \leq l \leq m\} \cup \{x_l : 1 \leq l \leq m-1\}$, where w_l is the edge corresponding to vv_l and x_l is the edge corresponding to $v_l v_{l+1}$. $V[L(F_{1,m})] = E(F_{1,m}) = \{v, v_l, u_l, u_l^{'} : 1 \leq l \leq m\} \cup \{u_l^{'} : 1 \leq l \leq m-1\}$.
For $1 \leq l \leq m$

- Each edge vv_l is subdivided by u_l of $F_{1,m}$
- Each edge v_lv_{l+1} is subdivided by $u_l^{'}$ of $F_{1,m}$

Let us prove the lower bound by the contradiction method. In $\chi_\rho[L(F_{1,m})]$, we need to prove four cases. So, we assume $\chi_\rho[L(F_{1,m})] < \frac{5m+4}{4}, \frac{5m+3}{4}, \frac{5m+2}{4}, \frac{5m+1}{4}$ for $m \equiv 0\ mod\ 4, m \equiv 1\ mod\ 4, m \equiv 2\ mod\ 4, m \equiv 3\ mod\ 4$ respectively. For that, we assume $\chi_\rho[L(F_{1,m})] < \frac{5m+4}{4}$, $\frac{5m+3}{4}$, $\frac{5m+2}{4}$, $\frac{5m+1}{4}$ for $m \equiv 0\ mod\ 4$, $m \equiv 1\ mod\ 4$, $m \equiv 2\ mod\ 4$, $m \equiv 3\ mod\ 4$ respectively. And select the colors for each valid vertex in $L(F_{1,m})$ as follows $\frac{5m}{4}$, $\frac{5m-1}{4}$, $\frac{5m-2}{4}$, $\frac{5m-3}{4}$ for $m \equiv 0\ mod\ 4$, $m \equiv 1\ mod\ 4$, $m \equiv 2\ mod\ 4$, $m \equiv 3\ mod\ 4$ respectively. By the definition line graph of fan graph, we have observed that $c(u_{2l}^{'}) = 1$ and $d(u_l, u_l^{'}) = 1, d(u_l, u_{l+1}^{'}) = 2$ and $d(u_{4l-3}^{'}, u_{4l+1}^{'}) = 3$. Here the maximum distance is 3 and the diameter of $\chi_\rho[L(F_{1,m})]$ is also 3. This shows that we can repeat only the colors 1 and 2 in $\chi_\rho[L(F_{1,m})]$. Since, we assumed $\frac{5m}{4}$, $\frac{5m-1}{4}$, $\frac{5m-2}{4}$, $\frac{5m-3}{4}$ for $m \equiv 0\ mod\ 4$, $m \equiv 1\ mod\ 4$, $m \equiv 2\ mod\ 4$, $m \equiv 3\ mod\ 4$ respectively. Therefore, we are left with a remaining colors $\frac{5m-8}{4}$, $\frac{5m-9}{4}$, $\frac{5m-10}{4}$, $\frac{5m-11}{4}$ for $m \equiv 0\ mod\ 4$, $m \equiv 1\ mod\ 4$, $m \equiv 2\ mod\ 4$, $m \equiv 3\ mod\ 4$ respectively. According to the definition of packing coloring, if two vertices of color i are at distance of at least $i+1$ apart and $d(u_l, u_{l+1}^{'}) = 2$ then $c(u_l) \neq c(u_{l+1}^{'})$ and the following colors are required $\frac{5m-8}{4}$, $\frac{5m-9}{4}$, $\frac{5m-10}{4}$, $\frac{5m-11}{4}$ for $m \equiv 0\ mod\ 4$, $m \equiv 1\ mod\ 4$, $m \equiv 2\ mod\ 4$, $m \equiv 3\ mod\ 4$ respectively for each u_l and $u_{l+1}^{'}$. While this proves a contradictory value compared to the desired output, the statement $\chi_\rho[L(F_{1,m})] < \frac{5m+4}{4}$, $\frac{5m+3}{4}$, $\frac{5m+2}{4}$, $\frac{5m+1}{4}$ for $m \equiv 0\ mod\ 4$, $m \equiv 1\ mod\ 4$, $m \equiv 2\ mod\ 4$, $m \equiv 3\ mod\ 4$ respectively are wrong.
Hence, we conclude that the lower bound

$$\chi_\rho[L(F_{1,m})] \geq \begin{cases} \frac{5m+4}{4}\ if\ m \equiv 0\ mod\ 4 \\ \frac{5m+3}{4}\ if\ m \equiv 1\ mod\ 4 \\ \frac{5m+2}{4}\ if\ m \equiv 2\ mod\ 4 \\ \frac{5m+1}{4}\ if\ m \equiv 3\ mod\ 4 \end{cases}$$

Next, we need to calculate the upper bound

$$\chi_\rho[L(F_{1,m})] \leq \begin{cases} \frac{5m+4}{4}\ if\ m \equiv 0\ mod\ 4 \\ \frac{5m+3}{4}\ if\ m \equiv 1\ mod\ 4 \\ \frac{5m+2}{4}\ if\ m \equiv 2\ mod\ 4 \\ \frac{5m+1}{4}\ if\ m \equiv 3\ mod\ 4 \end{cases}$$

Consider the color function,
$V(L(F_{1,m})) \to \{1, 2, 3, ..., p\}$ is defined as follows,

$$\text{where } p = \begin{cases} \frac{5m+4}{4}\ if\ m \equiv 0\ mod\ 4 \\ \frac{5m+3}{4}\ if\ m \equiv 1\ mod\ 4 \\ \frac{5m+2}{4}\ if\ m \equiv 2\ mod\ 4 \\ \frac{5m+1}{4}\ if\ m \equiv 3\ mod\ 4 \end{cases}$$

$$\begin{array}{ll} c(u_{2l}^{'}) = 1 & \text{for } 1 \leq l \leq m-1 \\ c(u_{4l-3}^{'}) = 2 & \text{for } 1 \leq l \leq m-1 \\ c(u_{4l-1}^{'}) = l+2 & \text{for } 1 \leq l \leq m-1 \\ c(u_l) = 1 & \text{for } l = 1 \end{array}$$

If $m \equiv 0\ mod\ 4$

$$c(u_{l+1}) = \frac{m+4l+8}{4} \qquad \text{for } 1 \leq l \leq m$$

If $m \equiv 1\ mod\ 4$

$$c(u_{l+1}) = \frac{m+4l+7}{4} \qquad \text{for } 1 \leq l \leq m$$

If $m \equiv 2\ mod\ 4$

$$c(u_{l+1}) = \frac{m+4l+6}{4} \qquad \text{for } 1 \leq l \leq m$$

If $m \equiv 3\ mod\ 4$

$$c(u_{l+1}) = \frac{m+4l+5}{4} \qquad \text{for } 1 \leq l \leq m$$

Therefore, the upper bound of

$$\chi_\rho[L(F_{1,m})] \leq \begin{cases} \frac{5m+4}{4} & if\ m \equiv 0\ mod\ 4 \\ \frac{5m+3}{4} & if\ m \equiv 1\ mod\ 4 \\ \frac{5m+2}{4} & if\ m \equiv 2\ mod\ 4 \\ \frac{5m+1}{4} & if\ m \equiv 3\ mod\ 4 \end{cases}$$

Hence, $\chi_\rho[L(F_{1,m})] = \begin{cases} \frac{5m+4}{4} & if\ m \equiv 0\ mod\ 4 \\ \frac{5m+3}{4} & if\ m \equiv 1\ mod\ 4 \\ \frac{5m+2}{4} & if\ m \equiv 2\ mod\ 4 \\ \frac{5m+1}{4} & if\ m \equiv 3\ mod\ 4 \end{cases}$ □

3 Packing coloring of jump graph of certain graphs

Theorem 3.1. *If $n \geq 5$, then the packing chromatic number of the jump graph of path graph is $\chi_\rho[J(P_n)] = n - 2$.*

Proof. Let us assume $V(P_n) = \{v_l : 1 \leq l \leq n\}$ and $V[J(P_n)] = \{e_l : 1 \leq l \leq n-1\}$.
For $1 \leq l \leq n$

- e_l is the vertex corresponding to the edge v_lv_{l+1} of P_n

Let us prove the lower bound of $\chi_\rho[J(P_n)]$ by the contradiction method. For that, we assume that $\chi_\rho[J(P_n)] < n-2$. Now, select the colors for each valid vertex in $\chi_\rho[J(P_n)]$ as $n-3$. From jump graph of path graph, we observed $c(e_1) = c(e_2) = c_1$ and $d(e_1, e_3) = 1, d(e_l, e_{l+1}) = 2$ are true. Here, the maximum distance is 2 then the diameter of $J(P_n)$ is also 2. This shows that we can repeat only the color 1 as much as possible in $J(P_n)$. Since, we assumed $n-3$ colors for $J(P_n)$, now, we are left with a remaining colors as $n-4$. According to the definition of packing coloring, if two vertices of color i are at distance of at least $i+1$ apart and $d(e_l, e_{l+1}) = 2$ then $c(e_l) \neq c(e_{l+1})$ and $n-4$ colors are required for each e_l and e_{l+1}. While this proves a contradictory value compared to the desired output, the statement $\chi_\rho[J(P_n)] < n-2$ is wrong. Hence, we conclude that the lower bound $\chi_\rho[J(P_n)] \geq n-2$.
Next, we need to calculate the upper bound $\chi_\rho[J(P_n)] \leq n-2$.
On considering the color function $c : V[J(P_n)] \to \{c_1, c_2, ..., c_{n-2}\}$ is defined by,

$$\begin{aligned} &c(e_1) = c(e_2) = 1 \\ &c(e_l) = l-1 && \text{for } 3 \leq l \leq n-1 \end{aligned}$$

Therefore, the upper bound $\chi_\rho[J(P_n)] \leq n-2$. Hence, $\chi_\rho[J(P_n)] = n-2$. □

Theorem 3.2. *If $n \geq 5$, then the packing chromatic number of the jump graph of cycle graph is $\chi_\rho[J(C_n)] = n - 1$.*

Proof. Let us assume $V(C_n) = \{v_l : 1 \leq l \leq n\}$ and $V[J(C_n)] = \{e_l : 1 \leq l \leq n\}$.
For $1 \leq l \leq n$

- e_n is the vertex corresponding to the edge v_lv_n of C_n

For $1 \leq l \leq n-1$

- e_l is the vertex corresponding to the edge v_lv_{l+1} of C_n

Let us prove the lower bound of $\chi_\rho[J(C_n)]$ by the contradiction method. For that, we assume that $\chi_\rho[J(C_n)] < n-1$. Now, select the colors for each valid vertex in $\chi_\rho[J(C_n)]$ as $n-2$. From jump graph of cycle graph, we observed $c(e_1) = c(e_2) = c_1$, $d(e_1, e_3) = 1$ and $d(e_l, e_{l+1}) = 2$ are true. Here, the maximum distance is 2 then the diameter of $J(C_n)$ is also 2. This shows that we can repeat only the color 1 as much as possible in $J(C_n)$. Since, we assumed $n-2$ colors for $J(C_n)$, now, we are left with a remaining colors as $n-3$. According to the definition of packing coloring, if two vertices of color i are at distance of atleast $i+1$ apart and $d(e_l, e_{l+1}) = 2$ then $c(e_l) \neq c(e_{l+1})$ and $n-3$ colors are required for each e_l and e_{l+1}. While this proves a contradictory value compared to the desired output, the statement $\chi_\rho[J(C_n)] < n-1$ is wrong. Hence, we conclude that the lower bound $\chi_\rho[J(P_n)] \geq n-1$. Next, we need to calculate the

upper bound $\chi_\rho[J(C_n)] \leq n-1$.
On considering the color function $c : V[J(C_n)] \to \{c_1, c_2, ..., c_{n-1}\}$ is defined by,

$$\begin{array}{ll} c(e_1) = c(e_2) = 1 & \\ c(e_l) = l-1 & \text{for } 3 \leq l \leq n \end{array}$$

Therefore, the upper bound $\chi_\rho[J(C_n)] \leq n-1$.
Hence, $\chi_\rho[J(C_n)] = n-1$. □

Theorem 3.3. *If $n \geq 4$, then the packing chromatic number of the jump graph of wheel graph is $\chi_\rho[J(W_n)] = n+1$.*

Proof. Let us assume $V(W_n) = \{v, a_l : 1 \leq l \leq n\}$ and
$V[J(W_n)] = \{d_l, e_l : 1 \leq l \leq n\}$.
For $1 \leq l \leq n$

- d_n is the vertex corresponding to the edge $a_n a_1$ of W_n
- e_l is the vertex corresponding to the edge va_l of W_n

For $1 \leq l \leq n-1$

- d_l is the vertex corresponding to the edge $a_l a_{l+1}$ of W_n

Let us prove the lower bound of $\chi_\rho[J(W_n)]$ by the contradiction method. For that, we assume that $\chi_\rho[J(W_n)] < n+1$. Now, select the colors for each valid vertex in $\chi_\rho[J(W_n)]$ as n. From jump graph of wheel graph, we observed $c(e_l) = c_1$, $d(d_1, e_3) = 1$ and $d(e_l, d_{l+1}) = 2$ are true. Here, the maximum distance is 2 then the diameter of $J(W_n)$ is also 2. This shows that we can repeat only the color 1 as much as possible in $J(W_n)$. Since, we assumed n colors for $J(W_n)$, now, we are left with a remaining colors as $n-1$. According to the definition of packing coloring, if two vertices of color i are at distance of atleast $i+1$ apart and $d(e_l, d_{l+1}) = 2$ then $c(e_l) \neq c(d_{l+1})$ and $n-1$ colors are required for each e_l and d_{l+1}. While this proves a contradictory value compared to the desired output, the statement $\chi_\rho[J(W_n)] < n+1$ is .
Hence, we conclude that the lower bound $\chi_\rho[J(W_n)] \geq n+1$.
Next, we need to calculate the upper bound $\chi_\rho[J(W_n)] \leq n+1$.
On considering the color function $c : V[J(W_n)] \to \{c_1, c_2, ..., c_{n+1}\}$ is defined by,

$$\begin{array}{ll} c(e_l) = 1 & \text{for } 1 \leq l \leq n \\ c(d_l) = l+1 & \text{for } 1 \leq l \leq n \end{array}$$

Therefore, the upper bound $\chi_\rho[J(W_n)] \leq n+1$.
Hence, $\chi_\rho[J(W_n)] = n+1$. □

Theorem 3.4. *If $n \geq 3$, then the packing chromatic number of the jump graph of double wheel graph is $\chi_\rho[J(DW_n)] = 2n+1$.*

Proof. Let us assume that $V(DW_n) = \{v, a_l, b_l : 1 \leq l \leq n\}$ and $V[J(DW_n)] = \{d_l, e_l, f_l, g_l : 1 \leq l \leq n\}$.
For $1 \leq l \leq n$

- e_l is the vertex corresponding to the edge va_l of DW_n
- f_l is the vertex corresponding to the edge vb_l of DW_n
- d_n is the vertex corresponding to the edge $a_n a_1$ of DW_n
- g_n is the vertex corresponding to the edge $g_n g_1$ of DW_n

For $1 \leq l \leq n-1$

- d_l is the vertex corresponding to the edge $a_l a_{l+1}$ of DW_n
- g_l is the vertex corresponding to the edge $b_l b_{l+1}$ of DW_n

Let us prove the lower bound of $\chi_\rho[J(DW_n)]$ by the contradiction method. For that, we assume that $\chi_\rho[J(DW_n)] < 2n+1$. Now, select the colors for each valid vertex in $\chi_\rho[J(DW_n)]$ as $2n$. From jump graph of double wheel graph, we observed $c(e_l) = c(f_l) = c_1$, $d(d_1, e_3) = 1$ and $d(f_l, g_l) = 2$ are true. Here, the maximum distance is 2 then the diameter of $J(DW_n)$ is also 2. This shows that we can repeat only the color 1 as much as possible in $J(DW_n)$. Since, we assumed $2n$ colors for $J(DW_n)$, now, we are left with a remaining colors as $2n-1$. According to the definition of packing coloring, if two vertices of color i are at distance of atleast $i+1$ apart and $d(f_l, g_l) = 2$ then $c(f_l) \neq c(g_l)$ and $2n-1$ colors are required for each f_l and g_l. While this proves a contradictory value compared to the desired output, the statement $\chi_\rho[J(DW_n)] < 2n+1$ is wrong. Hence, we conclude that the lower bound $\chi_\rho[J(DW_n)] \geq 2n+1$.
Next, we need to calculate the upper bound $\chi_\rho[J(DW_n)] \leq 2n+1$.
On considering the color function $c : V[J(DW_n)] \to \{c_1, c_2, ..., c_{2n+1}\}$ is defined by,

$$
\begin{aligned}
&c(e_l) = c(f_l) = 1 && \text{for } 1 \leq l \leq n \\
&c(d_l) = l+1 && \text{for } 1 \leq l \leq n \\
&c(g_l) = n+1+l && \text{for } 1 \leq l \leq n
\end{aligned}
$$

Therefore, the upper bound $\chi_\rho[J(Dw_n)] \leq 2n+1$.
Hence, $\chi_\rho[J(DW_n)] = 2n+1$. □

Theorem 3.5. *If $n \geq 4$, then the packing chromatic number of the jump graph of helm graph is $\chi_\rho[J(H_n)] = 2n+1$.*

Proof. Let us assume that $V(H_n) = \{v, a_l, f_l : 1 \leq l \leq n\}$ and $V[J(H_n)] = \{d_l, e_l, g_l : 1 \leq l \leq n\}$.
For $1 \leq l \leq n$

- e_l is the vertex corresponding to the edge va_l of H_n
- g_l is the vertex corresponding to the edge $a_l f_l$ of H_n
- d_n is the vertex corresponding to the edge $a_n a_1$ of H_n

For $1 \leq l \leq n-1$

- d_l is the vertex corresponding to the edge $a_l a_{l+1}$ of H_n

Let us prove the lower bound of $\chi_\rho[J(H_n)]$ by the contradiction method. For that, we assume that $\chi_\rho[J(H_n)] < 2n+1$. Now, select the colors for each valid vertex in $\chi_\rho[J(H_n)]$ as $2n$. From jump graph of helm graph, we observed $c(e_l) = c_1$, $d(g_l, g_{l+1}) = 1$ and $d(e_l, d_l) = 2$ are true. Here, the maximum distance is 2 then the diameter of $J(H_n)$ is also 2. This shows that we can repeat only the color 1 as much as possible in $J(H_n)$. Since, we assumed $2n$ colors for $J(H_n)$, now, we are left with a remaining colors as $2n-1$. According to the definition of packing coloring, if two vertices of color i are at distance of atleast $i+1$ apart and $d(e_l, d_l) = 2$ then $c(e_l) \neq c(d_l)$ and $(2n-1)$ colors are required for each e_l and d_l. While this proves a contradictory value compared to the desired output, the statement $\chi_\rho[J(H_n)] < 2n+1$ is wrong. Hence, we conclude that the lower bound $\chi_\rho[J(H_n)] \geq 2n+1$.
Next, we need to calculate the upper bound $\chi_\rho[J(H_n)] \leq 2n+1$.
On considering the color function $c : V[J(H_n)] \to \{c_1, c_2, ..., c_{2n+1}\}$ is defined by,

$$
\begin{aligned}
&c(e_l) = 1 && \text{for } 1 \leq l \leq n \\
&c(d_l) = l+1 && \text{for } 1 \leq l \leq n \\
&c(g_l) = n+1+l && \text{for } 1 \leq l \leq n
\end{aligned}
$$

Therefore, the upper bound $\chi_\rho[J(H_n)] \leq 2n+1$.
Hence, $\chi_\rho[J(H_n)] = 2n+1$. □

Theorem 3.6. *If $n \geq 3$, then the packing chromatic number of the jump graph of closed helm graph is $\chi_\rho[J(CH_n)] = 3n+1$.*

Proof. Let us assume that $V(CH_n) = \{v, a_l, f_l : 1 \leq l \leq n\}$ and $V[J(CH_n)] = \{d_l, e_l, g_l, b_l : 1 \leq l \leq n\}$.
For $1 \leq l \leq n$

- e_l is the vertex corresponding to the edge va_l of CH_n
- b_l is the vertex corresponding to the edge a_lf_l of CH_n
- d_n is the vertex corresponding to the edge a_na_1 of CH_n
- g_n is the vertex corresponding to the edge f_na_1 of CH_n

For $1 \leq l \leq n-1$

- d_l is the vertex corresponding to the edge a_la_{l+1} of CH_n
- g_l is the vertex corresponding to the edge f_lf_{l+1} of CH_n

Let us prove the lower bound of $\chi_\rho[J(CH_n)]$ by the contradiction method. For that, we assume that $\chi_\rho[J(H_n)] < 3n+1$. Now, select the colors for each valid vertex in $\chi_\rho[J(CH_n)]$ as $3n$. From jump graph of closed helm graph, we observed $c(e_l) = c_1$, $d(e_l, b_l) = 1$ and $d(e_l, d_l) = 2$ are true. Here, the maximum distance is 2 then the diameter of $J(CH_n)$ is also 2. This shows that we can repeat only the color 1 as much as possible in $J(CH_n)$. Since, we assumed $3n$ colors for $J(CH_n)$, now, we are left with a remaining colors as $3n-1$. According to the definition of packing coloring, if two vertices of color i are at distance of at least $i+1$ apart and $d(e_l, d_l) = 2$ then $c(e_l) \neq c(d_l)$ and $3n-1$ colors are required for each e_l and d_l. While this proves a contradictory value compared to the desired output, the statement $\chi_\rho[J(CH_n)] < 3n+1$ is wrong. Hence, we conclude that the lower bound $\chi_\rho[J(CH_n)] \geq 3n+1$.
Next, we need to calculate the upper bound $\chi_\rho[J(CH_n)] \leq 3n+1$.
On considering the color function $c : V[J(CH_n)] \to \{c_1, c_2, ..., c_{3n+1}\}$ is defined by,

$c(e_l) = 1$	for $1 \leq l \leq n$
$c(d_l) = l+1$	for $1 \leq l \leq n$
$c(b_l) = n+1+l$	for $1 \leq l \leq n$
$c(g_l) = n+4+l$	for $1 \leq l \leq n$

Therefore, the upper bound $\chi_\rho[J(CH_n)] \leq 3n+1$. Hence, $\chi_\rho[J(CH_n)] = 3n+1$. □

Theorem 3.7. *If $n \geq 3$, then the packing chromatic number of the jump graph of sunlet graph is $\chi_\rho[J(S_n)] = 2n-2$.*

Proof. Let us assume that $V(S_n) = \{a_l, f_l : 1 \leq l \leq n\}$ and $V[J(S_n)] = \{e_l, b_l : 1 \leq l \leq n\}$.
For $1 \leq l \leq n$

- b_l is the vertex corresponding to the edge a_lf_l of S_n
- e_n is the vertex corresponding to the edge a_na_1 of S_n

For $1 \leq l \leq 2n-4$

- e_l is the vertex corresponding to the edge a_la_{l+1} of S_n

Let us prove the lower bound of $\chi_\rho[J(S_n)]$ by the contradiction method. For that, we assume that $\chi_\rho[J(S_n)] < 2n-2$. Now, select the colors for each valid vertex in $\chi_\rho[J(S_n)]$ as $2n-3$. From jump graph of sunlet graph, we observed $c(e_1) = c(e_2) = c_1$, $d(b_l, b_{l+1}) = 1$ and $d(e_1, b_1) = 2$ are true. Here, the maximum distance is 2 then the diameter of $J(S_n)$ is also 2. This shows that we can repeat only the color 1 as much as possible in $J(S_n)$. Since, we assumed $2n-3$ colors for $J(S_n)$, now, we are left with a remaining colors as $2n-4$. According to the definition of packing coloring, if two vertices of color i are at distance of atleast $i+1$ apart and $d(e_1, b_1) = 2$ then $c(e_1) \neq c(b_1)$ and $2n-4$ colors are required for each e_1 and b_1. While this proves a contradictory value compared to the desired output, the statement $\chi_\rho[J(S_n)] < 2n-2$ is wrong. Hence, we conclude that the lower bound $\chi_\rho[J(S_n)] \geq 2n-2$. Next, we need to calculate the upper bound $\chi_\rho[J(S_n)] \leq 2n-2$.
On considering the color function $c : V[J(S_n)] \to \{c_1, c_2, ..., c_{2n-2}\}$ is defined by,

$c(e_1) = c(e_2) = c(b_1) = 1$	for $1 \leq l \leq n$
$c(e_l) = l-1$	for $3 \leq l \leq n$
$c(b_l) = c(e_n) + l - 1$	for $2 \leq l \leq n$

Therefore, the upper bound $\chi_\rho[J(S_n)] \leq 2n-2$.
Hence, $\chi_\rho[J(S_n)] = 2n-2$.

□

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