Palestine Journal of Mathematics

Vol 10 (Special Issue II, 2021), 41–45

ON $^*g\alpha$ - INTERIOR AND $^*g\alpha$ - CLOSURE IN NEUTROSOPHIC TOPOLOGICAL SPACES

A. R. Nivetha and M. Vigneshwaran

Communicated by Saeid Jafari

MSC 2010 Classifications: 06D72, 03E72.

Keywords and phrases: $\mathcal{N}_{*g\alpha}$-interior, $\mathcal{N}_{*g\alpha}$ - closure, $\mathcal{N}_{*g\alpha}$ - neighbourhood point, $\mathcal{N}_{*g\alpha}$ - frontier and $\mathcal{N}_{*g\alpha}$ - border.

Abstract In this article, we has mainly focused on the conception of $\mathcal{N}_{*g\alpha}$-interior, $\mathcal{N}_{*g\alpha}$ - closure and $\mathcal{N}_{*g\alpha}$ - neighbourhood point in neutrosophic topological spaces. Consequently, using the concept of $\mathcal{N}_{*g\alpha}$-interior, $\mathcal{N}_{*g\alpha}$ - closure we also proposed $\mathcal{N}_{*g\alpha}$ - frontier, $\mathcal{N}_{*g\alpha}$ - border. Further establishes their properties and investigate the relation between $\mathcal{N}_{*g\alpha}$ - frontier and $\mathcal{N}_{*g\alpha}$ - border.

1 Introduction

The denotation of neutrosophic set was commenced by Smarandache[13] and clarified that the neutrosophic set is a generalization of intuitionistic fuzzy set (IFS). Salama and Alblowi[10] proposed the conception of neutrosophic topological space (NTS) in 2012 that had been investigated recently. All the elements within neutrosophic set have the degree of membership, indefiniteness and degree of non-membership values. Arokiarani et al.[2] introduced the α-closed set in (NTS). The fundamental set like semi/pre/α - open sets are defined in (NTS) and investigated by many mathematicians[[4],[9]]. Dhavaseelan and Saied Jafari[3] introduced neutrosophic generalized closed sets in 2017. Sreeja et al.[14] studied the denotation of neutrosophic gα-closed sets and neutrosophic gα-open sets in (NTS). Vigneshwaran et al.[15] deïňĄned a new closed set as $^*g\alpha$-closed sets in topological spaces which has been applied to define some topological functions as continuous functions, irresolute functions and homeomorphic functions with some separable axioms. Recently the connotation of $\mathcal{N}_{*g\alpha}$-CS in (NTS) are implemented and discussed by Nivetha et.al.[8]. The basic definitions, that are utilized in the consecutive section are refered from the references [[1], [2], [3], [5], [6], [7], [8], [10], [11], [12], [14]].
Throughout this paper neutrosophic $^*g\alpha$-interior, neutrosophic $^*g\alpha$-closure, neutrosophic $^*g\alpha$-neighbourhood, neutrosophic $^*g\alpha$-frontier and neutrosophic $^*g\alpha$-border is denoted by $\mathcal{N}_{*g\alpha}$-I^*, $\mathcal{N}_{*g\alpha}$-C^*, $\mathcal{N}_{*g\alpha}$-N, $\mathcal{N}_{*g\alpha}$-BR and $\mathcal{N}_{*g\alpha}$-FR respectively.

2 Properties of $\mathcal{N}_{*g\alpha}$-I^* and $\mathcal{N}_{*g\alpha}$-C^*

Definition 2.1 A subset $\mathcal{S}$ of $(\mathcal{W}, \varsigma)$ is known as $\mathcal{N}_{*g\alpha}$-I^* if $\mathcal{N}_{*g\alpha}$-$I^*(\mathcal{S}) = \bigcup\{\mathcal{R}$: $\mathcal{R}$ is $\mathcal{N}_{*g\alpha}$-OS and $\mathcal{R} \subset \mathcal{S}\}$.

Definition 2.2 A subset $\mathcal{S}$ of $(\mathcal{W}, \varsigma)$ is known as $\mathcal{N}_{*g\alpha}$-C^* if $\mathcal{N}_{*g\alpha}$-$C^*(\mathcal{S}) = \bigcap\{\mathcal{R}$: $\mathcal{R}$ is $\mathcal{N}_{*g\alpha}$-CS and $\mathcal{S} \subset \mathcal{R}\}$.

Definition 2.3 A subset $\mathcal{S}$ of $(\mathcal{W}, \varsigma)$ is said to be $\mathcal{N}_{*g\alpha}$-N of $s \in \mathcal{S}$ if there exists a $\mathcal{N}_{*g\alpha}$-OS $\mathcal{R}$ such that $s \in \mathcal{R} \subset \mathcal{S}$.

Definition 2.4 A point $x \in \mathcal{S}$ of NTS $(\mathcal{W}, \varsigma)$ is said to be $\mathcal{N}_{*g\alpha}$-I^*point of $\mathcal{S}$ if $\mathcal{S}$ is $\mathcal{N}_{*g\alpha}$-N of x.

Theorem 2.1. *If $\mathcal{S}$ and $\mathcal{R}$ be any two subsets of NTS $(\mathcal{W},\varsigma)$. Then, $\mathcal{N}_{*g\alpha}$-I^*($\mathcal{W}$) = $\mathcal{W}$ and $\mathcal{N}_{*g\alpha}$-$I^*(\Phi)$ = Φ.*

Proof. Since $\mathcal{W}$ and Φ are $\mathcal{N}_{*g\alpha}$-OS, then $\mathcal{N}_{*g\alpha}$-$I^*(\mathcal{W})$ = $\bigcup\{\mathcal{R} : \mathcal{R}$ is $\mathcal{N}_{*g\alpha}$-OS and $\mathcal{R} \subset \mathcal{W}\}$ = $\mathcal{W} \cup \{\mathcal{R}$ is $\mathcal{N}_{*g\alpha}$-$OS\}$ = $\mathcal{W}$. Thus $\mathcal{N}_{*g\alpha}$-$I^*(\mathcal{W})$ = $\mathcal{W}$. Consequently, $\mathcal{N}_{*g\alpha}$-$I^*(\Phi)$= Φ, since there is no $\mathcal{N}_{*g\alpha}$ other than Φ contained in Φ.□

Theorem 2.2. *Let $\mathcal{S}$ and $\mathcal{R}$ be any two subsets of NTS $(\mathcal{W},\varsigma)$. Then,*
(i) If $\mathcal{R}$ is any $\mathcal{N}_{*g\alpha}$-OS contained in $\mathcal{S}$, then $\mathcal{R} \subset \mathcal{N}_{*g\alpha}$-$I^*(\mathcal{S})$.
(ii) If $\mathcal{S} \subset \mathcal{R}$, then $\mathcal{N}_{*g\alpha}$-$I^*(\mathcal{S}) \subset \mathcal{N}_{*g\alpha}$-$I^*(\mathcal{R})$.
(iii) $\mathcal{N}_{*g\alpha}$-$I^*(\mathcal{N}_{*g\alpha}$-$I^*(\mathcal{S})) = \mathcal{N}_{*g\alpha}$-$I^*(\mathcal{S})$.

Proof. (i) Let $\mathcal{R}$ be any $\mathcal{N}_{*g\alpha}$-OS, such that $\mathcal{R} \subset \mathcal{S}$. Let $x \in \mathcal{S}$. Since $\mathcal{S}$ is a $\mathcal{N}_{*g\alpha}$-OS contained in $\mathcal{S}$. x is a $\mathcal{N}_{*g\alpha}$-interior point of $\mathcal{S}$. That is $x \in \mathcal{N}_{*g\alpha}$-$I^*(\mathcal{S})$. Hence $\mathcal{R} \in \mathcal{N}_{*g\alpha}$-$I^*(\mathcal{S})$.

(ii) Let $\mathcal{S}$ and $\mathcal{R}$ be any two subsets of NTS $(\mathcal{W},\varsigma)$ such that $\mathcal{S} \subset \mathcal{R}$. Let $x \in \mathcal{N}_{*g\alpha}$-$I^*(\mathcal{S})$. Then x is a $\mathcal{N}_{*g\alpha}$-interior point of $\mathcal{S}$ and so $\mathcal{S}$ is a $\mathcal{N}_{*g\alpha}$-neighbourhood of x. Since $\mathcal{R} \supset \mathcal{S}$, $\mathcal{R}$ is also a $\mathcal{N}_{*g\alpha}$-neighbourhood of x. This implies that $x \in \mathcal{N}_{*g\alpha}$-$I^*(\mathcal{R})$. Thus we have shown that $x \in \mathcal{N}_{*g\alpha}$-$I^*(\mathcal{S})$ implies $x \in \mathcal{N}_{*g\alpha}$-$I^*(\mathcal{R})$. Hence $\mathcal{N}_{*g\alpha}$-$I^*(\mathcal{S}) \subset \mathcal{N}_{*g\alpha}$-$I^*(\mathcal{R})$.

(iii) Let $\mathcal{S}$ be any subset of $(\mathcal{W},\varsigma)$. By definition, $\mathcal{N}_{*g\alpha}$-$I^*(\mathcal{S})$ is $\mathcal{N}_{*g\alpha}$-OS and hence $\mathcal{N}_{*g\alpha}$-$I^*(\mathcal{N}_{*g\alpha}$-$I^*(\mathcal{S})) = \mathcal{N}_{*g\alpha}$-$I^*(\mathcal{S})$.□

Theorem 2.3. *If a subset $\mathcal{S}$ of a NTS $(\mathcal{W},\varsigma)$ is $\mathcal{N}_{*g\alpha}$-OS, then $\mathcal{N}_{*g\alpha}$-$I^*(\mathcal{S})$= $\mathcal{S}$.*

Proof. Let $\mathcal{S}$ be a $\mathcal{N}_{*g\alpha}$-OS subset of $(\mathcal{W},\varsigma)$. We know that $\mathcal{N}_{*g\alpha}$-$I^*(\mathcal{S}) \subset \mathcal{S}$. Also, $\mathcal{M}$ is $\mathcal{N}_{*g\alpha}$-OS contained in $\mathcal{S}$. We know that if $\mathcal{M}$ is any $\mathcal{N}_{*g\alpha}$-OS contained in $\mathcal{S}$, then $\mathcal{M} \subset \mathcal{N}_{*g\alpha}$-$I^*(\mathcal{S})$, we have $\mathcal{S} \subset \mathcal{N}_{*g\alpha}$-$I^*(\mathcal{S})$. Hence $\mathcal{N}_{*g\alpha}$-$I^*(\mathcal{S})$ = $\mathcal{S}$.□

Theorem 2.4. *Let $\mathcal{S}$ and $\mathcal{R}$ be any two subsets of NTS $(\mathcal{W},\varsigma)$. Then,*
(i) $\mathcal{N}_{*g\alpha}$-C^*($\mathcal{W}$) = $\mathcal{W}$ and $\mathcal{N}_{*g\alpha}$-C^*(Φ) = Φ.
(ii) If $\mathcal{R}$ is any $\mathcal{N}_{*g\alpha}$-CS containing $\mathcal{S}$, then $\mathcal{R} \supset \mathcal{N}_{*g\alpha}$-$C^*(\mathcal{S})$.
(iii) If $\mathcal{S} \subset \mathcal{R}$, then $\mathcal{N}_{*g\alpha}$-$C^*(\mathcal{S}) \subset \mathcal{N}_{*g\alpha}$-$C^*(\mathcal{R})$.
(iv) $\mathcal{N}_{*g\alpha}$-$C^*(\mathcal{N}_{*g\alpha}$-$C^*(\mathcal{S})) = \mathcal{N}_{*g\alpha}$-$C^*(\mathcal{S})$.

Proof. (i) By the definition of $\mathcal{N}_{*g\alpha}$-C^*, $\mathcal{W}$ is the only $\mathcal{N}_{*g\alpha}$-CS containing $\mathcal{W}$. Therefore $\mathcal{N}_{*g\alpha}$-$C^*(\mathcal{W})$ = Intersection of all the $\mathcal{N}_{*g\alpha}$-CS containing $\mathcal{W}$. Thus is $\mathcal{N}_{*g\alpha}$-$C^*(\mathcal{W})$ = $\mathcal{W}$.Consequently, $\mathcal{N}_{*g\alpha}$-$C^*(\Phi)$ = Intersection of all the $\mathcal{N}_{*g\alpha}$-CS containing Φ. Thus $\mathcal{N}_{*g\alpha}$-$C^*(\Phi)$ = Φ.

(ii) Let $\mathcal{S} \subset \mathcal{R}$, Where $\mathcal{R}$ be any $\mathcal{N}_{*g\alpha}$-CS. Since $\mathcal{N}_{*g\alpha}$-$C^*(\mathcal{S})$ is the intersection of all $\mathcal{N}_{*g\alpha}$-CS containing $\mathcal{S}$, $\mathcal{N}_{*g\alpha}$-$C^*(\mathcal{S})$ is contained in every $\mathcal{N}_{*g\alpha}$-CS contains $\mathcal{S}$. Hence in particular $\mathcal{N}_{*g\alpha}$-$C^*(\mathcal{S}) \subset \mathcal{R}$.

(iii) Let $\mathcal{S}$ and $\mathcal{R}$ be any two subsets of NTS $(\mathcal{W},\varsigma)$. Since $\mathcal{S} \subset \mathcal{R}$. If $\mathcal{R} \subset \mathcal{H}$, then $\mathcal{N}_{*g\alpha}$-$C^*(\mathcal{R}) \subset \mathcal{H}$.Since $\mathcal{S} \subset \mathcal{R} \subset \mathcal{H} \in \mathcal{N}_{*g\alpha}$-CS, we have $\mathcal{N}_{*g\alpha}$-$C^*(\mathcal{S}) \subset \mathcal{H}$. Therefore $\mathcal{N}_{*g\alpha}$-$C^*(\mathcal{S}) \subset \cap\{\mathcal{H} : \mathcal{R} \subset \mathcal{H} \in \mathcal{N}_{*g\alpha}$-$CS\}$ = $\mathcal{N}_{*g\alpha}$-$C^*(\mathcal{R})$. Hence $\mathcal{N}_{*g\alpha}$-$C^*(\mathcal{S}) \subset \mathcal{N}_{*g\alpha}$-$C^*(\mathcal{R})$.

(iv) Let $\mathcal{S} \subset \mathcal{R}$ be a $\mathcal{N}_{*g\alpha}$-CS.Then by the definition, $\mathcal{N}_{*g\alpha}$-$C^*(\mathcal{S}) \subset \mathcal{R}$. Since $\mathcal{N}_{*g\alpha}$-$C^*(\mathcal{S}) \subset \mathcal{R} \subset \mathcal{N}_{*g\alpha}$-$CS$ containing $\mathcal{S}$. It follows that $\mathcal{N}_{*g\alpha}$-$C^*(\mathcal{N}_{*g\alpha}$-$C^*(\mathcal{S})) \subset \mathcal{N}_{*g\alpha}$-$C^*(\mathcal{S})$. Therefore $\mathcal{N}_{*g\alpha}$-$C^*(\mathcal{N}_{*g\alpha}$-$C^*(\mathcal{S}))$ = $\mathcal{N}_{*g\alpha}$-$C^*(\mathcal{S})$.□

Theorem 2.5. *If a subset $\mathcal{S}$ of a NTS $(\mathcal{W}, \varsigma)$ is $\mathcal{N}_{*g\alpha}$-CS, then it is a $\mathcal{N}_{*g\alpha}$-$C^*(\mathcal{S})$.*

Proof. Let $\mathcal{S}$ be $\mathcal{N}_{*g\alpha}$-CS of $(\mathcal{W}, \varsigma)$. We know that $\mathcal{S} \subset \mathcal{N}_{*g\alpha}$-$C^*(\mathcal{S})$. Also, a set is itself subset and $\mathcal{S}$ is $\mathcal{N}_{*g\alpha}$-CS. Thus, we have $\mathcal{N}_{*g\alpha}$-$C^*(\mathcal{S}) \subset \mathcal{S}$. Hence $\mathcal{N}_{*g\alpha}$-$C^*(\mathcal{S}) = \mathcal{S}$.□

3 $\mathcal{N}_{*g\alpha} - BR$ and $\mathcal{N}_{*g\alpha} - FR$

Definition 3.1 For any subset $\mathcal{S}$ of $(\mathcal{W}, \varsigma)$, the $\mathcal{N}_{*g\alpha} - BR$ of $\mathcal{S}$ is defined by $\mathcal{N}_{*g\alpha}[BR(\mathcal{S})] = \mathcal{S} \backslash \mathcal{N}_{*g\alpha} - I^*(\mathcal{S})$.
Definition 3.2 For any subset $\mathcal{S}$ of $(\mathcal{W}, \varsigma)$, the $\mathcal{N}_{*g\alpha} - FR$ of $\mathcal{S}$ is defined by $\mathcal{N}_{*g\alpha}[FR(\mathcal{S})] = \mathcal{N}_{*g\alpha} - C^*(\mathcal{S}) \backslash \mathcal{N}_{*g\alpha} - I^*(\mathcal{S})$.

Theorem 3.1. *In the NTS $(\mathcal{W}, \varsigma)$, for any subset $\mathcal{S}$ of $\mathcal{W}$, the following statements are hold.*
(i) $\mathcal{N}_{*g\alpha}[BR(\Phi)] = \mathcal{N}_{*g\alpha}[BR(\mathcal{W})] = \Phi$.
(ii) $\mathcal{S} = \mathcal{N}_{*g\alpha} - I^*(\mathcal{S}) \cup \mathcal{N}_{*g\alpha}[BR(\mathcal{S})]$.
(iii) $\mathcal{N}_{*g\alpha} - I^*(\mathcal{S}) \cap \mathcal{N}_{*g\alpha}[BR(\mathcal{S})] = \Phi$.
(iv) $\mathcal{N}_{*g\alpha} - I^*(\mathcal{S}) = \mathcal{S} \backslash \mathcal{N}_{*g\alpha}[BR(\mathcal{S})]$.
(v) $\mathcal{N}_{*g\alpha} - I^*(\mathcal{N}_{*g\alpha}[BR(\mathcal{S})]) = \Phi$.
(vi) $\mathcal{S}$ *is* $\mathcal{N}_{*g\alpha}$-*OS iff* $\mathcal{N}_{*g\alpha}[BR(\mathcal{S})] = \Phi$.
(vii) $\mathcal{N}_{*g\alpha}[BR(\mathcal{N}_{*g\alpha} - I^*(\mathcal{S})) = \Phi$.
(viii) $\mathcal{N}_{*g\alpha}[BR(\mathcal{N}_{*g\alpha}[BR(\mathcal{S})])] = \mathcal{N}_{*g\alpha}[BR(\mathcal{S})]$.
(ix) $N_{*g\alpha}[BR(\mathcal{S})] = \mathcal{S} \cap \mathcal{N}_{*g\alpha} - C^*(\mathcal{W} \backslash \mathcal{S})$.

Proof. Statements (i) to (iv) are obvious by the definition of $\mathcal{N}_{*g\alpha}[BR(\mathcal{S})]$. To prove (v): If possible, let us assume $x \in \mathcal{N}_{*g\alpha} - I^*(\mathcal{N}_{*g\alpha}[BR(\mathcal{S})])$. Then $x \in \mathcal{N}_{*g\alpha}[BR(\mathcal{S})]$, since $\mathcal{N}_{*g\alpha}[BR(\mathcal{S})] \subseteq \mathcal{S}$, $x \in \mathcal{N}_{*g\alpha} - I^*(\mathcal{N}_{*g\alpha}[BR(\mathcal{S})]) \subseteq \mathcal{N}_{*g\alpha} - I^*(\mathcal{S})$. Therefore $x \in \mathcal{N}_{*g\alpha} - I^*(\mathcal{S}) \cap \mathcal{N}_{*g\alpha}[BR(\mathcal{S})]$, which is the contradiction to (iii). Hence (v) is proved. $\mathcal{S}$ is $\mathcal{N}_{*g\alpha} - OS$ iff $\mathcal{N}_{*g\alpha} - I^*(\mathcal{S}) = \mathcal{S}$. But $\mathcal{N}_{*g\alpha}[BR(\mathcal{S})] = \mathcal{S} \backslash \mathcal{N}_{*g\alpha} - I^*(\mathcal{S})$ implies $\mathcal{N}_{*g\alpha}[BR(\mathcal{S})] = \Phi$. This proves (vi) and (vii). When $\mathcal{S} = \mathcal{N}_{*g\alpha}[BR(\mathcal{S})]$, then we have $\mathcal{N}_{*g\alpha}[BR(\mathcal{N}_{*g\alpha}[BR(\mathcal{S})])] = \mathcal{N}_{*g\alpha}[BR(\mathcal{S})] \backslash \mathcal{N}_{*g\alpha} - I^*(\mathcal{N}_{*g\alpha}[BR(\mathcal{S})])$. By using (ii), we get the proof of (viii). Now, $\mathcal{N}_{*g\alpha}[BR(\mathcal{S})] = \mathcal{S} \backslash \mathcal{N}_{*g\alpha} - I^*(\mathcal{S}) = \mathcal{S} \cap (\mathcal{W} \backslash \mathcal{N}_{*g\alpha} - I^*(\mathcal{S})) = \mathcal{S} \cap \mathcal{N}_{*g\alpha} - C^*(\mathcal{W} \backslash \mathcal{S})$.□

Theorem 3.2. *In the NTS $(\mathcal{W}, \varsigma)$, for any subset $\mathcal{S}$ of $\mathcal{W}$, the following statements are hold.*
(i) $\mathcal{N}_{*g\alpha}[FR(\Phi)] = \mathcal{N}_{*g\alpha}[FR(\mathcal{W})] = \Phi$.
(ii) $\Phi = \mathcal{N}_{*g\alpha} - I^*(\mathcal{S}) \cap \mathcal{N}_{*g\alpha}[FR(\mathcal{S})]$.
(iii) $\mathcal{N}_{*g\alpha}[FR(\mathcal{S})] \subseteq \mathcal{N}_{*g\alpha} - C^*(\mathcal{S})$.
(iv) $\mathcal{N}_{*g\alpha} - I^*(\mathcal{S}) \cup \mathcal{N}_{*g\alpha}[FR(\mathcal{S})] = \mathcal{N}_{*g\alpha} - C^*(\mathcal{S})$.
(v) $\mathcal{N}_{*g\alpha} - I^*(\mathcal{S}) = \mathcal{S} \backslash \mathcal{N}_{*g\alpha}[FR(\mathcal{S})]$.
(vi) If $\mathcal{S}$ *is* $\mathcal{N}_{*g\alpha}$-*CS, then* $\mathcal{S} = \mathcal{N}_{*g\alpha} - I^*(\mathcal{S}) \cup \mathcal{N}_{*g\alpha}[FR(\mathcal{S})]$.
(vii) $\mathcal{N}_{*g\alpha}FR(\mathcal{S} = \mathcal{N}_{*g\alpha}FR(\mathcal{N}_{*g\alpha}[FR(\mathcal{S})])$.

Proof. Statements (i) to (vii) are true by the definition of $\mathcal{N}_{*g\alpha}[FR(\mathcal{S})]$.□

4 Relation between $\mathcal{N}_{*g\alpha} - BR$ and $\mathcal{N}_{*g\alpha} - FR$

Theorem 4.1. *In the NTS $(\mathcal{W}, \varsigma)$, for any subset $\mathcal{S}$ of $\mathcal{W}$, the following statements are hold.*
(i) $\mathcal{N}_{*g\alpha}[BR(\mathcal{S})] \backslash \mathcal{N}_{*g\alpha}[FR(\mathcal{S})] = \Phi$.

(ii) $\mathcal{N}_{*g\alpha}[BR(\mathcal{S})] \subseteq \mathcal{N}_{*g\alpha}[FR(\mathcal{S})]$.
(iii) $\mathcal{N}_{*g\alpha}[FR(\mathcal{N}_{*g\alpha}[BR(\mathcal{S})])] = \mathcal{N}_{*g\alpha}[BR(\mathcal{S})]$.
(iv) $\mathcal{N}_{*g\alpha}[BR(\mathcal{N}_{*g\alpha}[FR(\mathcal{S})])] = \mathcal{N}_{*g\alpha}[FR(\mathcal{S})]$.
(v) If $\mathcal{S}$ *is* $\mathcal{N}_{*g\alpha} - OS$*, then* $\mathcal{N}_{*g\alpha}[FR(\mathcal{S})] \cup \mathcal{N}_{*g\alpha}[BR(\mathcal{S})] = \mathcal{N}_{*g\alpha}[FR(\mathcal{S})]$.
(vi) $\mathcal{N}_{*g\alpha}[FR(\mathcal{S})] \cap \mathcal{N}_{*g\alpha}[BR(\mathcal{S})] = \mathcal{N}_{*g\alpha}[BR(\mathcal{S})]$.
(vii) $\overline{\mathcal{N}_{*g\alpha}[FR(\mathcal{S})]} \cup \overline{\mathcal{N}_{*g\alpha}[BR(\mathcal{S})]} = \overline{\mathcal{N}_{*g\alpha}[BR(\mathcal{S})]}$.
(viii) $\overline{\mathcal{N}_{*g\alpha}[FR(\mathcal{S})]} \cap \overline{\mathcal{N}_{*g\alpha}[BR(\mathcal{S})]} = \overline{\mathcal{N}_{*g\alpha}[FR(\mathcal{S})]}$.

Proof. Statement (i)to (iv) are obvious by the definitions of $\mathcal{N}_{*g\alpha}[FR(\mathcal{S})]$ and $\mathcal{N}_{*g\alpha}[BR(\mathcal{S})]$. Since $\mathcal{S}$ is $\mathcal{N}_{*g\alpha} - OS$, then we know that, $\mathcal{N}_{*g\alpha}[BR(\mathcal{S})] = \Phi$ which implies $\mathcal{N}_{*g\alpha}[FR(\mathcal{S})] \cup \Phi = \mathcal{N}_{*g\alpha}[FR(\mathcal{S})]$. Hence (v) is proved. We know, $\mathcal{N}_{*g\alpha}[BR(\mathcal{S})] \subseteq \mathcal{N}_{*g\alpha}[FR(\mathcal{S})]$. Thus $\mathcal{N}_{*g\alpha}[FR(\mathcal{S})] \cap \mathcal{N}_{*g\alpha}[BR(\mathcal{S})] = \mathcal{N}_{*g\alpha}[BR(\mathcal{S})]$. It proves (vi). By taking compliment and using De Morgan's law to the (vi) implies $\overline{\mathcal{N}_{*g\alpha}[FR(\mathcal{S})]} \cup \overline{\mathcal{N}_{*g\alpha}[BR(\mathcal{S})]} = \overline{\mathcal{N}_{*g\alpha}[BR(\mathcal{S})]}$, it gives the proof of (vii). Similarly we can prove the statement (viii).□

References

[1] Al-Omeri W and Jafari S, On Generalized Closed Sets and Generalized Pre Closed Sets in Neutrosophic Topological Spaces, *Mathematics*, **7(1)**, 1–12 (2018).

[2] Arokiarani I, Dhavaseelan R, Jafari S and Parimala M, On some new notions and functions in neutrosophic topological spaces,*Neutrosophic Sets Systems*, **16**, 16–19 (2017).

[3] Dhavaseelan R and Jafari S, Generalized neutrosophic closed sets, *In New Trends in Neutrosophic Theory and Application*, **2**, 261–273 (2018).

[4] Ishwarya P and Bageerathi K, On Neutrosophic semi-open sets in Neutrosophic topological spaces, *International Journal of Mathematics Trends and Technology*, **37(3)**, 214–223 (2016).

[5] Ishwarya P and Bageerathi K, A Study on Neutrosophic Frontier and Neutrosophic Semi-Frontier in Neutrosophic Topological Spaces, *Neutrosophic Sets and Systems*, **16(3)**, 6–15 (2017).

[6] Jayanthi D, α Generalized Closed Sets in Neutrosophic Topological Spaces, *International Journal of Mathematics Trends and Technology*, 88–91 (2018).

[7] Mohammed Ali Jaffer and Ramesh K, Neutrosophic Generalized Pre Regular Closed Sets, *Neutrosophic Sets and Systems*, **30(13)**, 171–181 (2019).

[8] Nivetha A R and Vigneshwaran M, On $\mathcal{N}_{*g\alpha}$-closed sets in neutrosophic topological spaces(Communicated).

[9] Qays Hatem Imran, Smarandache et. al, On Neutrosophic semi alpha open sets, *Neutrosophic sets and systems*, **18(5)**, 37–42 (2017).

[10] Salama A A and Alblowi S A, Neutrosophic Set and Neutrosophic Topological Spaces, *IOSR Journal of Mathematics*, **3(4)**, 31–35 (2012).

[11] Shanthi V K, Chandrasekar S, Safina Begam K, Neutrosophic Generalized Semi Closed Sets In Neutrosophic Topological Spaces, *International Journal of Research in Advent Technology*, **6(7)**, 1739–1743 (2018).

[12] Smarandache F, *Neutrosophy and Neutrosophic Logic*, First International Conference on Neutrosophy, Neutrosophic Logic, Set, Probability and Statistics, University of New Mexico, Gallup, NM, USA, (2002).

[13] Smarandache F, *A Unifying Field in Logics: Neutrosophic Logic, Neutrosophy, Neutrosophic Set, Neutrosophic Probability*, American Research Press, Rehoboth, NM, USA, (1999).

[14] Sreeja D and Sarankumar T, Generalized Alpha Closed in Neutrosophic topological spaces, *Journal of Applied Science and Computations*, **5(11)**, 1816–1823 (2018).

[15] Vigneshwaran M and Devi R, On $G_{\alpha o}$-kernel in the digital plane, *International Journal of Mathematical Archieve*, **3(6)**, 2358–2373 (2012).

Author information

A. R. Nivetha, PG and Research Department of Mathematics, Kongunadu Arts and Science College, Coimbatore - 641 029, Tamil Nadu, India.
E-mail: arnivetha@kongunaducollege.ac.in

M. Vigneshwaran, PG and Research Department of Mathematics, Kongunadu Arts and Science College, Coimbatore - 641 029, Tamil Nadu, India.
E-mail: vignesh.mat@gmail.com

Received : January 5, 2021
Accepted : April 5, 2021