ON $^*g\alpha$ - INTERIOR AND $^*g\alpha$ - CLOSURE IN NEUTROSOPHIC TOPOLOGICAL SPACES

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Abstract In this article, we has mainly focused on the conception of $N^*g\alpha$-interior, $N^*g\alpha$ - closure and $N^*g\alpha$ - neighbourhood point in neutrosophic topological spaces. Consequently, using the concept of $N^*g\alpha$-interior, $N^*g\alpha$- closure we also proposed $N^*g\alpha$ - frontier, $N^*g\alpha$ - border. Further establishes their properties and investigate the relation between $N^*g\alpha$ - frontier and $N^*g\alpha$ - border.

1 Introduction

The denotation of neutrosophic set was commenced by Smarandache[13] and clarified that the neutrosophic set is a generalization of intuitionistic fuzzy set (IFS). Salama and Alblowi[10] proposed the conception of neutrosophic topological space (NTS) in 2012 that had been investigated recently. All the elements within neutrosophic set have the degree of membership, indefiniteness and degree of non-membership values. Arokiarani et al.[2] introduced the $\alpha$-closed set in (NTS). The fundamental set like semi/pre/$\alpha$ - open sets are defined in (NTS) and investigated by many mathematicians[[4],[9]]. Dhavaseelan and Saied Jafari[3] introduced neutrosophic generalized closed sets in 2017. Sreeja et al.[14] studied the denotation of neutrosophic $g\alpha$-closed sets and neutrosophic $g\alpha$-open sets in (NTS). Vigneshwaran et al.[15] defined a new closed set as $^*g\alpha$-closed sets in topological spaces which has been applied to define some topological functions as continuous functions, irresolute functions and homeomorphic functions with some separable axioms. Recently the connotation of $N^*g\alpha$-CS in (NTS) are implemented and discussed by Nivetha et.al.[8]. The basic definitions, that are utilized in the consecutive section are refered from the references [[1], [2], [3], [5], [6], [7], [8], [10], [11], [12], [14]].

Throughout this paper neutrosophic $N^*g\alpha$-I* and $N^*g\alpha$-C* are studied.

2 Properties of $N^*g\alpha$-I* and $N^*g\alpha$-C*

Definition 2.1 A subset $S$ of $(\mathcal{W}, \varsigma)$ is known as $N^*g\alpha$-I* if $N^*g\alpha$-I*(S) = $\bigcup \{ R : R$ is $N^*g\alpha$-OS and $R \subset S \}.$

Definition 2.2 A subset $S$ of $(\mathcal{W}, \varsigma)$ is known as $N^*g\alpha$-C* if $N^*g\alpha$-C*(S) = $\bigcap \{ R : R$ is $N^*g\alpha$-CS and $S \subset R \}.$

Definition 2.3 A subset $S$ of $(\mathcal{W}, \varsigma)$ is said to be $N^*g\alpha$-N of $s \in S$ if there exists a $N^*g\alpha$-OS $R$ such that $s \in R \subset S.$

Definition 2.4 A point $x \in S$ of NTS $(\mathcal{W}, \varsigma)$ is said to be $N^*g\alpha$I*point of $S$ if $S$ is $N^*g\alpha$-N of $x.$
Theorem 2.1. If $S$ and $R$ be any two subsets of NTS $(W, \varsigma)$. Then, $N_{ga}^{-}I^{+}(W) = W$ and $N_{ga}^{-}I^{+}(\Phi) = \Phi$.

Proof. Since $W$ and $\Phi$ are $N_{ga}^{-}$OS, then $N_{ga}^{-}I^{+}(W) = \{ R : R$ is $N_{ga}^{-}$OS and $R \subset W \} \cup \{ R$ is $N_{ga}^{-}$OS $\} = W$. Thus $N_{ga}^{-}I^{+}(W) = W$. Consequently, $N_{ga}^{-}I^{+}(\Phi) = \Phi$, since there is no $N_{ga}^{-}$ other than $\Phi$ contained in $\Phi$. □

Theorem 2.2. Let $S$ and $R$ be any two subsets of NTS $(W, \varsigma)$. Then,

(i) If $R$ is any $N_{ga}^{-}$OS contained in $S$, then $R \subset N_{ga}^{-}I^{+}(S)$.

(ii) If $S \subset R$, then $N_{ga}^{-}I^{+}(S) \subset N_{ga}^{-}I^{+}(R)$.

(iii) $N_{ga}^{-}I^{+}(N_{ga}^{-}I^{+}(S)) = N_{ga}^{-}I^{+}(S)$.

Proof. (i) Let $R$ be any $N_{ga}^{-}$OS, such that $R \subset S$. Let $x \in S$. Since $S$ is a $N_{ga}^{-}$OS contained in $S$, $x$ is an $N_{ga}^{-}$OS point of $S$. Hence $R \subset N_{ga}^{-}I^{+}(S)$.

(ii) Let $S$ and $R$ be any two subsets of NTS $(W, \varsigma)$ such that $S \subset R$. Let $x \in N_{ga}^{-}I^{+}(S)$. Then $x$ is an $N_{ga}^{-}$OS point of $S$ and so $S$ is an $N_{ga}^{-}$OS of $x$. Since $R \supset S$, $R$ is also any $N_{ga}^{-}$OS of $x$. This implies that $x \in N_{ga}^{-}I^{+}(R)$. Thus we have shown that $x \in N_{ga}^{-}I^{+}(S)$ implies $x \in N_{ga}^{-}I^{+}(R)$. Hence $N_{ga}^{-}I^{+}(S) \subset N_{ga}^{-}I^{+}(R)$.

(iii) Let $S$ be any subset of $(W, \varsigma)$. By definition, $N_{ga}^{-}I^{+}(S)$ is $N_{ga}^{-}$OS and hence $N_{ga}^{-}I^{+}(N_{ga}^{-}I^{+}(S)) = N_{ga}^{-}I^{+}(S)$. □

Theorem 2.3. If a subset $S$ of a NTS $(W, \varsigma)$ is $N_{ga}^{-}$OS, then $N_{ga}^{-}I^{+}(S) = S$.

Proof. Let $S$ be a $N_{ga}^{-}$OS subset of $(W, \varsigma)$. We know that $N_{ga}^{-}I^{+}(S) \subset S$. Also, $M$ is $N_{ga}^{-}$OS contained in $S$. We know that if $M$ is any $N_{ga}^{-}$OS contained in $S$, then $M \subset N_{ga}^{-}I^{+}(S)$, we have $S \subset N_{ga}^{-}I^{+}(S)$. Hence $N_{ga}^{-}I^{+}(S) = S$. □

Theorem 2.4. Let $S$ and $R$ be any two subsets of NTS $(W, \varsigma)$. Then,

(i) $N_{ga}^{-}C^{+}(W) = W$ and $N_{ga}^{-}C^{+}(\Phi) = \Phi$.

(ii) If $R$ is any $N_{ga}^{-}$CS containing $S$, then $R \supset N_{ga}^{-}C^{+}(S)$.

(iii) If $S \subset R$, then $N_{ga}^{-}C^{+}(S) \subset N_{ga}^{-}C^{+}(R)$.

(iv) $N_{ga}^{-}C^{+}(N_{ga}^{-}C^{+}(S)) = N_{ga}^{-}C^{+}(S)$.

Proof. (i) By the definition of $N_{ga}^{-}C^{+}$, $W$ is the only $N_{ga}^{-}$CS containing $W$. Therefore $N_{ga}^{-}C^{+}(W) = N_{ga}^{-}C^{+}(\Phi) = \Phi$.

(ii) Let $S \subset R$, Where $R$ be any $N_{ga}^{-}$CS. Since $N_{ga}^{-}C^{+}(S)$ is the intersection of all $N_{ga}^{-}$CS containing $S$, $N_{ga}^{-}C^{+}(S)$ is contained in every $N_{ga}^{-}C^{+}$ contains $S$. Hence in particular $N_{ga}^{-}C^{+}(S) \subset R$.

(iii) Let $S$ and $R$ be any two subsets of NTS $(W, \varsigma)$. Since $S \subset R$. If $R \subset H$, then $N_{ga}^{-}C^{+}(R) \subset H$. Since $S \subset R \subset H \in N_{ga}^{-}$CS, we have $N_{ga}^{-}C^{+}(S) \subset H$. Therefore $N_{ga}^{-}C^{+}(S) \subset \cap \{ H : R \subset H \in N_{ga}^{-}$CS $\} = N_{ga}^{-}C^{+}(R)$. Hence $N_{ga}^{-}C^{+}(S) \subset N_{ga}^{-}C^{+}(R)$.

(iv) Let $S \subset R$ be a $N_{ga}^{-}$CS. Then by the definition, $N_{ga}^{-}C^{+}(S) \subset R$. Since $N_{ga}^{-}C^{+}(S) \subset R \subset N_{ga}^{-}$CS containing $S$. It follows that $N_{ga}^{-}C^{+}(N_{ga}^{-}C^{+}(S)) \subset N_{ga}^{-}C^{+}(S)$. Therefore $N_{ga}^{-}C^{+}(N_{ga}^{-}C^{+}(S)) = N_{ga}^{-}C^{+}(S)$. □
Theorem 2.5. If a subset $S$ of a NTS $(W, \varsigma)$ is $N_{\gamma a}$-CS, then it is a $N_{\gamma a}$-$C^*(S)$.

**Proof.** Let $S$ be $N_{\gamma a}$-CS of $(W, \varsigma)$. We know that $S \in N_{\gamma a}$-$C^*(S)$. Also, a set is itself subset and $S$ is $N_{\gamma a}$-CS. Thus, we have $N_{\gamma a}$-$C^*(S) \subseteq S$. Hence $N_{\gamma a}$-$C^*(S) = S$. □

3 $N_{\gamma a} - BR$ and $N_{\gamma a} - FR$

**Definition 3.1** For any subset $S$ of $(W, \varsigma)$, the $N_{\gamma a} - BR$ of $S$ is defined by $N_{\gamma a}[BR(S)] = S \setminus N_{\gamma a} - I^*(S)$.

**Definition 3.2** For any subset $S$ of $(W, \varsigma)$, the $N_{\gamma a} - FR$ of $S$ is defined by $N_{\gamma a}[FR(S)] = N_{\gamma a} - C^*(S) \setminus N_{\gamma a} - I^*(S)$.

**Theorem 3.1.** In the NTS $(W, \varsigma)$, for any subset $S$ of $W$, the following statements hold.

(i) $N_{\gamma a}[BR(\Phi)] = N_{\gamma a}[BR(W)] = \Phi$.
(ii) $S = N_{\gamma a} - I^*(S) \cup N_{\gamma a}[BR(S)]$.
(iii) $N_{\gamma a} - I^*(S) \cap N_{\gamma a}[BR(S)] = \Phi$.
(iv) $N_{\gamma a} - I^*(S) = S \setminus N_{\gamma a}[BR(S)]$.
(v) $N_{\gamma a} - I^*(N_{\gamma a}[BR(S)]) = \Phi$.
(vi) $S$ is $N_{\gamma a}$-OS iff $N_{\gamma a}[BR(S)] = \Phi$.
(vii) $N_{\gamma a}[BR(N_{\gamma a}[BR(S)])] = N_{\gamma a}[BR(S)]$.
(viii) $N_{\gamma a}[BR(N_{\gamma a}[BR(S)])] = N_{\gamma a}[BR(S)]$.
(ix) $N_{\gamma a}[BR(S)] = S \cap N_{\gamma a} - C^*(W \setminus S)$.

**Proof.** Statements (i) to (iv) are obvious by the definition of $N_{\gamma a}[BR(S)]$. To prove (v): If possible, let us assume $x \in N_{\gamma a} - I^*(S) \cap N_{\gamma a}[BR(S)]$, then $x \in N_{\gamma a}[BR(S)]$, since $N_{\gamma a}[BR(S)] \subseteq S$, $x \in N_{\gamma a} - I^*(N_{\gamma a}[BR(S)]) \subseteq N_{\gamma a} - I^*(S)$. Therefore $x \in N_{\gamma a} - I^*(S) \cap N_{\gamma a}[BR(S)]$, which is the contradiction to (iii). Hence (v) is proved. $S$ is $N_{\gamma a} - OS$ iff $N_{\gamma a} - I^*(S) \cap N_{\gamma a}[BR(S)] = \Phi$. But $N_{\gamma a}[BR(S)] = S \setminus N_{\gamma a} - I^*(S)$ implies $N_{\gamma a}[BR(S)] = \Phi$. This proves (vi) and (viii). Now, $N_{\gamma a}[BR(S)] = S \setminus N_{\gamma a} - I^*(S) = S \cap (W \setminus N_{\gamma a} - I^*(S))$.

**Theorem 3.2.** In the NTS $(W, \varsigma)$, for any subset $S$ of $W$, the following statements hold.

(i) $N_{\gamma a}[FR(\Phi)] = N_{\gamma a}[FR(W)] = \Phi$.
(ii) $\Phi = N_{\gamma a} - I^*(S) \cap N_{\gamma a}[FR(S)]$.
(iii) $N_{\gamma a}[FR(S)] \subseteq N_{\gamma a} - C^*(S)$.
(iv) $N_{\gamma a} - I^*(S) \cap N_{\gamma a}[FR(S)] = N_{\gamma a} - C^*(S)$.
(v) $N_{\gamma a} - I^*(S) = S \setminus N_{\gamma a}[FR(S)]$.
(vi) If $S$ is $N_{\gamma a}$-CS, then $S = N_{\gamma a} - I^*(S) \cup N_{\gamma a}[FR(S)]$.
(vii) $N_{\gamma a}[FR(S) = N_{\gamma a}[FR(N_{\gamma a}[FR(S)])]$.

**Proof.** Statements (i) to (vii) are true by the definition of $N_{\gamma a}[FR(S)]$. □

4 Relation between $N_{\gamma a} - BR$ and $N_{\gamma a} - FR$

**Theorem 4.1.** In the NTS $(W, \varsigma)$, for any subset $S$ of $W$, the following statements hold.

(i) $N_{\gamma a}[BR(S)] \setminus N_{\gamma a}[FR(S)] = \Phi$.
(ii) $N_{ga}(BR(S)) \subseteq N_{ga}(FR(S))$.

(iii) $N_{ga}(FR(N_{ga}(BR(S)))) = N_{ga}(BR(S))$.

(iv) $N_{ga}(BR(N_{ga}(FR(S)))) = N_{ga}(FR(S))$.

(v) If $S$ is $N_{ga} - OS$, then $N_{ga}(BR(S)) \cup N_{ga}(BR(S)) = N_{ga}(FR(S))$.

(vi) $N_{ga}(FR(S)) \cap N_{ga}(BR(S)) = N_{ga}(BR(S))$.

(vii) $N_{ga}(FR(S)) \cup N_{ga}(BR(S)) = N_{ga}(FR(S))$.

(viii) $N_{ga}(FR(S)) \cap N_{ga}(BR(S)) = N_{ga}(FR(S))$.

**Proof.** Statement (i) to (iv) are obvious by the definitions of $N_{ga}(FR(S))$ and $N_{ga}(BR(S))$. Since $S$ is $N_{ga} - OS$, then we know that, $N_{ga}(BR(S)) = \Phi$ which implies $N_{ga}(FR(S)) \cup \Phi = N_{ga}(FR(S))$. Hence (v) is proved. We know, $N_{ga}(BR(S)) \subseteq N_{ga}(FR(S))$. Thus $N_{ga}(FR(S)) \cap N_{ga}(BR(S)) = N_{ga}(BR(S))$. It proves (vi). By taking compliment and using De Morgan’s law to the (vi) implies $N_{ga}(FR(S)) \cup N_{ga}(BR(S)) = N_{ga}(BR(S))$, it gives the proof of (vii). Similarly we can prove the statement (viii). □

**References**


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