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Distributions of waves in rotating double walled graphene tubule with effect of thermo-magnetic force using shell theory

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Abstract This paper is concerned with the distribution of waves in double walled graphene tubule which is stimulated with the elastic shell theory. The governing equation of motion for a graphene tubule under the effect of thermo-magnetic force is derived. The dispersed connection of spinning graphene tubule with thermo magnetic reaction is achieved in light of outward power, Lorentz magnetic force and thermal force. The impact of progress in temperature, rotational and magnetic on the properties of vibrations are talked about. The quantitative estimation of dimensionless phase velocity is figured and represented as scattered bends for symmetric and asymmetric methods of vibrations. Also, dispersed bends of graphene tubule acquired by barring the thermal, magnetic and rotating force are outlined and analogized with the genuine writing. The effect of powers on the dissemination of waves in the graphene tubule might be valuable for future plans in nano mechanical frameworks, micro electromechanical and its applications.

1 Introduction

Lijima [1] discovered the carbon nanotubes in mid 1990s prompts colossal exploration interest in the field of nanotechnology because of its predominance in mechanical, electronic and thermal conductivity measure. Li et al. [2] discussed the impact of magnetic field on the vibration behavior of Multi Walled Carbon Nanotube (MWCNT) through vander waals force between two layer and applied Lorentz magnetic force on each layer, the coupled conditions are acquired and the outcomes shows that least recurrence of MWCNT diminishes nonlinearly with applied transverse attractive field while the most noteworthy recurrence stays unaltered. Sun and Zhang [3] investigated the material properties reliant on the length of plate structures, these outcomes demonstrate that discrete material structure at the nanoscale can't be homogeneized into a continuum model now the non-local elastic continuum models were considered in the investigation of nanostructure. Ponnusamy and Amuthalaksmi [4] and [5] investigates the dispersion analysis of thermo magnetic field of Double Walled Carbon Nanotube (DWCNTs) using nonlocal Timoshenko-beam model and also deals with dispersion analysis of thermo magnetic effect on double layered nanoplate embedded in an elastic medium. Nonlocal shell model for propagation of Carbon Nanotubes (CNTs) are discussed. Torkaman Asadi et al. [6] studied the free vibrations and stability of high speed stability.

Yang et al. [7] investigated the distribution of waves in fluid conveying Carbon Nanotubes (CNTs) using beam theory and gradient coupled theory. Cook et al. [8] discussed the frictions in rotating nanotubes. Safarpour et al. [9] deals with high speed rotating nanotube reinforced cyclindrical piezoelectrical shell. Mingo and Broido [10] studied the ballistics thermal conductance and its limits. Yu et al. [11] deals with thermal conductance and thermopower of individual Single Walled Carbon Nanotube(SWCNT). Xue [12] investigates the model for thermal conductivity of CNT.

In this paper, the effect of thermo-magnetic vibrations in rotating graphene tubule is explored in double walled graphene tubule. The wave engendering of turning graphene tubule with the impact of thermal, rotating and magnetic force is investigated utilizing shell model. The scattered connection is acquired for graphene tubule by deriving the equations of motion and the mathematical values are figured.

2 Formulation of the problem

Consider the graphene tubule under the thermal, magnetic and rotating force as shown in Figure 1. The transverse deformation behavior of each nested graphene tubule was assumed by vander waals interaction. In shell theory, applying some suspicion, such as shear twisting can be disre-

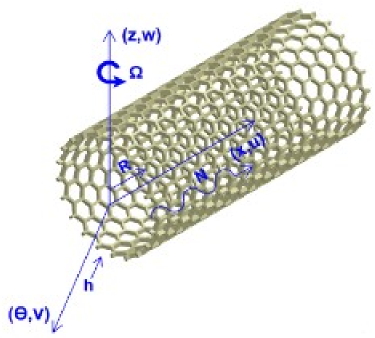

Figure 1. Coordinate system of rotating graphene tubule under the thermal, magnetic and rotating force

garded, the strain relocations of second and higher orders are ignored and for unreformed center surface the typical turns out to be straight line and for transformed it endures no expansion, accordingly the equation of motion for graphene tubule under the elastic shell theory is given by

$$\frac{\partial M_x}{\partial x} + \frac{1}{R}\frac{\partial M_{\theta x}}{\partial x} = \rho h \frac{\partial^2 u}{\partial t^2} \tag{2.1}$$

$$\frac{1}{R}\frac{\partial M_\theta}{\partial \theta} + \frac{\partial M_{x\theta}}{\partial x} = \rho h \frac{\partial^2 v}{\partial t^2} \tag{2.2}$$

$$\frac{M_\theta}{R} + q = \rho h \frac{\partial^2 w}{\partial t^2} \tag{2.3}$$

Where x is the coordination along the shell, θ is the polar angle, R is the radius of rotating graphene tubule, u, v and w are the decranation module of radial, longitudinal and tangential directions, ρ is the mass thickness, q is the preeminent force, h is the thickness of the shell and M_x and M_θ are normal stress, $M_{x\theta}$ and $M_{\theta x}$ are shear stress components. In consonance with Hooke's Law the stress components are derived as

$$M_x = \frac{Eh}{1-\nu^2}(\epsilon_x + \nu\epsilon_\theta) \tag{2.4}$$

$$M_\theta = \frac{Eh}{1-\nu^2}(\epsilon_\theta + \nu\epsilon_x) \tag{2.5}$$

$$M_{\theta x} = M_{x\theta} = \frac{Eh}{2(1+\nu)}\gamma \tag{2.6}$$

Where E and ν are tensile modulus and Poisson's coefficient, ϵ_x and ϵ_θ denotes the axial strains and γ denotes shear strain. The expression of axial strains and shear strain related to the displacement equations are given by

$$\epsilon_x = \frac{\partial u}{\partial x} \tag{2.7}$$

$$\epsilon_\theta = \frac{1}{R}(w + \frac{\partial v}{\partial \theta}) \tag{2.8}$$

$$\gamma = \frac{\partial v}{\partial x} + \frac{1}{R}\frac{\partial u}{\partial \theta} \tag{2.9}$$

Substituting Eqns (2.7) – (2.9) in Eqns (2.4) – (2.6), the normal stress and shear stress component in view of decranation is obtained as

$$M_x = \frac{Eh}{1-\nu^2}(\frac{\partial u}{\partial x} + \frac{\nu}{R}(w + \frac{\partial v}{\partial \theta})) \tag{2.10}$$

$$M_\theta = \frac{Eh}{1-\nu^2}(\frac{w}{R} + \frac{1}{R}\frac{\partial v}{\partial \theta} + \nu\frac{\partial u}{\partial x}) \tag{2.11}$$

$$M_{\theta x} = M_{x\theta} = \frac{Eh}{2(1+\nu)}(\frac{\partial v}{\partial x} + \frac{1}{R}\frac{\partial u}{\partial \theta}) \tag{2.12}$$

Further the axial force q in-terms of thermal and rotating force is given by

$$q = \frac{1}{h}(T(x) - N_t)\frac{\partial^2 w}{\partial x^2} \tag{2.13}$$

Where N_t denotes the thermal field and $T(x)$ denotes the revolving force[13] as follows

$$N_t = -\frac{EA}{1-2\nu}\alpha_x T \tag{2.14}$$

$$T(x) = -\frac{\rho A \Omega^2 L^2}{2} \tag{2.15}$$

Where T denotes the change in temperature, α_x denotes the amount of temperature extension, Ω denotes the revolving speed and L denotes the length. Assuming that the magnetic vulnerability η of graphene tubule approaches to magnetic vulnerability to the whole region in all directions, the magnetic force[14] in-terms of displacement equation is given by

$$f_x = 0, f_\theta = \eta A(\frac{H_x}{R^2}\frac{\partial^2 v}{\partial \theta^2} + H_x^2\frac{\partial^2 v}{\partial t^2}), f_z = \eta H_x^2 A\frac{\partial^2 w}{\partial x^2} \tag{2.16}$$

Where f_x, f_θ and f_z are the components of Lorentz force exerted on graphene tubule along x, θ and z-direction.
Substituting Eqns (2.10) – (2.16) in Eqns (2.1) – (2.3) , we get

$$\frac{Eh}{1-\nu^2}(\frac{\partial^2 u}{\partial x^2} + \frac{\nu}{R}(\frac{\partial w}{\partial x} + \frac{\partial^2 v}{\partial x \partial \theta})) + \frac{Eh}{2R(1+\nu)}(\frac{\partial^2 v}{\partial \theta \partial x} + \frac{1}{R}\frac{\partial^2 u}{\partial \theta^2}) + \frac{f_x}{h} = \rho h\frac{\partial^2 u}{\partial t^2} \tag{2.17}$$

$$\frac{Eh}{1-\nu^2}\frac{1}{R}(\frac{1}{R}\frac{\partial^2 v}{\partial \theta^2} + \nu\frac{\partial^2 u}{\partial \theta \partial x} + \frac{1}{R}\frac{\partial w}{\partial \theta}) + \frac{Eh}{2(1+\nu)}(\frac{1}{R}\frac{\partial^2 u}{\partial x \partial \theta} + \frac{\partial^2 v}{\partial x^2}) + \frac{f_\theta}{h} = \rho h\frac{\partial^2 v}{\partial t^2} \tag{2.18}$$

$$-\frac{Eh}{1-\nu^2}\frac{1}{R}(\frac{w}{R} + \frac{1}{R}\frac{\partial v}{\partial \theta} + \nu\frac{\partial u}{\partial x}) + q + \frac{f_z}{h} = \rho h\frac{\partial^2 w}{\partial t^2} \tag{2.19}$$

Eqns(2.17) – (2.19) represents the displacement equation of motion for single walled graphene tubule.

3 Formulation for the double walled graphene tubule

Consider the double walled graphene tubule subjected to thermo-magnetic and rotating force. Assuming the outer layer and inner layer of the graphene tubule with the effect of thermal, magnetic and rotating force, then the governing equation of motion for double walled graphene tubule is given by

$$\frac{Eh}{1-\nu^2}(\frac{\partial^2 u_1}{\partial x^2} + \frac{\nu}{R_1}(\frac{\partial w_1}{\partial x} + \frac{\partial^2 v_1}{\partial x \partial \theta})) + \frac{Eh}{2R_1(1+\nu)}(\frac{\partial^2 v_1}{\partial \theta \partial x} + \frac{1}{R_1}\frac{\partial^2 u_1}{\partial \theta^2}) + \frac{f_x}{h} = \rho h\frac{\partial^2 u_1}{\partial t^2} \tag{3.1}$$

$$\frac{Eh}{1-\nu^2}\frac{1}{R_1}(\frac{1}{R_1}\frac{\partial^2 v_1}{\partial\theta^2}+\nu\frac{\partial^2 u_1}{\partial\theta\partial x}+\frac{1}{R_1}\frac{\partial w_1}{\partial\theta})+\frac{Eh}{2(1+\nu)}(\frac{1}{R_1}\frac{\partial^2 u_1}{\partial x\partial\theta}+\frac{\partial^2 v_1}{\partial x^2})+\frac{f_\theta}{h}=\rho h\frac{\partial^2 v_1}{\partial t^2} \quad (3.2)$$

$$-\frac{Eh}{1-\nu^2}\frac{1}{R_1}(\frac{w_1}{R_1}+\frac{1}{R_1}\frac{\partial v_1}{\partial\theta}+\nu\frac{\partial u_1}{\partial x})+q_1+\frac{f_z}{h}=\rho h\frac{\partial^2 w_1}{\partial t^2} \quad (3.3)$$

$$\frac{Eh}{1-\nu^2}(\frac{\partial^2 u_2}{\partial x^2}+\frac{\nu}{R_2}(\frac{\partial w_2}{\partial x}+\frac{\partial^2 v_2}{\partial x\partial\theta}))+\frac{Eh}{2R_2(1+\nu)}(\frac{\partial^2 v_2}{\partial\theta\partial x}+\frac{1}{R_2}\frac{\partial^2 u_2}{\partial\theta^2})+\frac{f_x}{h}=\rho h\frac{\partial^2 u_2}{\partial t^2} \quad (3.4)$$

$$\frac{Eh}{1-\nu^2}\frac{1}{R_2}(\frac{1}{R_2}\frac{\partial^2 v_2}{\partial\theta^2}+\nu\frac{\partial^2 u_2}{\partial\theta\partial x}+\frac{1}{R_2}\frac{\partial w_2}{\partial\theta})+\frac{Eh}{2(1+\nu)}(\frac{1}{R_2}\frac{\partial^2 u_2}{\partial x\partial\theta}+\frac{\partial^2 v_2}{\partial x^2})+\frac{f_\theta}{h}=\rho h\frac{\partial^2 v_2}{\partial t^2} \quad (3.5)$$

$$-\frac{Eh}{1-\nu^2}\frac{1}{R_2}(\frac{w_2}{R_2}+\frac{1}{R_2}\frac{\partial v_2}{\partial\theta}+\nu\frac{\partial u_2}{\partial x})+q_2+\frac{f_z}{h}=\rho h\frac{\partial^2 w_2}{\partial t^2} \quad (3.6)$$

Where the axial force q in-terms of thermal, rotating force and vander waals interaction is given by

$$q_i=\frac{1}{h}p_i+\frac{1}{h}(T(x)-N_t)\frac{\partial^2 w_i}{\partial x^2} \quad (3.7)$$

in which p_i denotes transverse deformation due to vander waals interaction is given by $p_i=\sum_{j=1}^{2}c(w_i-w_j),\ (i\neq j)$.

4 Solution of the problem

Distribution of waves in graphene tubule under a extensive magnetic field, temperature and rotating force is studied by considering the harmonic solutions of waves u, v and w in the form

$$u_i=U_i\cos n\theta\exp i(\gamma x-\omega t) \quad (4.1)$$

$$v_i=V_i\sin n\theta\exp i(\gamma x-\omega t) \quad (4.2)$$

$$w_i=W_i\cos n\theta\exp i(\gamma x-\omega t) \quad (4.3)$$

consequently U_i, V_i and W_i denotes longitudinal, circumferential and radial amplitude of displacement, n, γ, and ω denotes the mode, wave number and frequency. Using Eqns $(4.1)-(4.3)$ and Eqn (3.7) in Eqns$(3.1)-(3.6)$ a set of homogeneous equation is obtained in the form

$$[-\gamma^2-\frac{1-\nu}{2R_1^2}n^2+\frac{\omega^2}{c_p^2}]U_1+[\frac{in\gamma(1+\nu)}{2R_1}]V_1+\frac{i\gamma\nu}{R_1}W_1=0 \quad (4.4)$$

$$[-\frac{1-\nu}{2R_1^2}n^2-\frac{\nu ni\gamma}{R_1}]U_1+[-\frac{n^2}{R_1^2}-\frac{1-\nu}{2}\gamma^2+\frac{\omega^2}{c_p^2}+\frac{1-\nu^2}{R_1^2Eh^2}\eta AH_x^2n^2-\frac{1-\nu^2}{Eh^2}\eta AH_x^2\gamma^2]V_1+\frac{n}{R_1^2}W_1=0 \quad (4.5)$$

$$[-\frac{\nu i\gamma}{R_1}]U_1+\frac{n}{R_1^2}V_1+[-\frac{1}{R_1^2}-\frac{1-\nu^2}{Eh^2}\gamma^2(T(x)-N_t-\eta AH_x^2)+\frac{\omega^2}{c_p^2}-\frac{1-\nu^2c}{Eh^2}]W_1+\frac{1-\nu^2c}{Eh^2}W_2=0 \quad (4.6)$$

$$[-\gamma^2-\frac{1-\nu}{2R_2^2}n^2+\frac{\omega^2}{c_p^2}]U_2+[\frac{in\gamma(1+\nu)}{2R_2}]V_2+\frac{i\gamma\nu}{R_2^2}W_2=0 \quad (4.7)$$

$$[-\frac{1-\nu}{2R_2^2}n^2-\frac{\nu ni\gamma}{R_2}]U_2+[-\frac{n^2}{R_2^2}-\frac{1-\nu}{2}\gamma^2+\frac{\omega^2}{c_p^2}+\frac{1-\nu^2}{R_2^2Eh^2}\eta AH_x^2n^2-\frac{1-\nu^2}{Eh^2}\eta AH_x^2\gamma^2]V_2+\frac{n}{R_1^2}W_2=0 \tag{4.8}$$

$$[-\frac{\nu i\gamma}{R_2}]U_2+\frac{n}{R_2}V_2+[-\frac{1}{R_2^2}-\frac{1-\nu^2}{Eh^2}\gamma^2(T(x)-N_t-\eta AH_x^2)+\frac{\omega^2}{c_p^2}-\frac{1-\nu^2c}{Eh^2}]W_2+\frac{1-\nu^2c}{Eh^2}W_1=0 \tag{4.9}$$

Establishing the following dimensionless parameters namely,
$\overline{c}=\frac{c}{c_p},\overline{\gamma}=h\gamma,\overline{\omega}=\frac{h\omega}{c_p},\overline{N}=\frac{N_t}{Eh^2},\overline{h}=\frac{h}{R},\overline{T(x)}=\frac{T(X)}{Eh^2},\overline{\eta}=\frac{\eta AH_x^2}{Eh^2},\varsigma=\frac{(1-\nu^2)c}{Eh^2}$
in Eqns (4.4) – (4.9), a set of homogeneous equations in dimensionless form is obtained in the form

$$[\overline{\omega}^2-\overline{\gamma}^2-\frac{(1-\nu)\overline{h}^2n^2}{2}]U_1+\frac{i(1+\nu)\overline{h}n\overline{\gamma}}{2}V_1+i\nu\overline{h}\overline{\gamma}W_1=0 \tag{4.10}$$

$$[\frac{i(1+\nu)\overline{\gamma}\overline{h}n}{2}]U_1+[\overline{h}^2n^2+\frac{(1-\nu)\overline{\gamma}^2}{2}+(1-\nu^2)\overline{\eta}(\overline{h}^2n^2+\overline{\gamma}^2)-\overline{\omega}^2]V_1+n\overline{h}^2W_1=0 \tag{4.11}$$

$$[i\nu\overline{h}\overline{\gamma}]U_1+n\overline{h}^2V_1+[\overline{\omega}^2+(1-\nu^2)\overline{\gamma}^2(\overline{N}-\overline{T(x)}-\overline{\eta})-\overline{h}^2-\varsigma]W_1+\varsigma W_2=0 \tag{4.12}$$

$$[\overline{\omega}^2-\overline{\gamma}^2-\frac{(1-\nu)\overline{h}^2n^2}{2}]U_2+\frac{i(1+\nu)\overline{h}n\overline{\gamma}}{2}V_2+i\nu\overline{h}\overline{\gamma}W_2=0 \tag{4.13}$$

$$[\frac{i(1+\nu)\overline{\gamma}\overline{h}n}{2}]U_2+[\overline{h}^2n^2+\frac{(1-\nu)\overline{\gamma}^2}{2}+(1-\nu^2)\overline{\eta}(\overline{h}^2n^2+\overline{\gamma}^2)-\overline{\omega}^2]V_2+n\overline{h}^2W_2=0 \tag{4.14}$$

$$[i\nu\overline{h}\overline{\gamma}]U_2+n\overline{h}^2V_2+[\overline{\omega}^2+(1-\nu^2)\overline{\gamma}^2(\overline{N}-\overline{T(x)}-\overline{\eta})-\overline{h}^2-\varsigma]W_2+\varsigma W_1=0 \tag{4.15}$$

The Eqns (4.10)-(4.15) can be written in the form of matrix,

$$\begin{bmatrix} \overline{\omega}^2-g_1 & g_2 & g_3 & 0 & 0 & 0 \\ g_2 & -(\overline{\omega}^2-g_4) & g_5 & 0 & 0 & 0 \\ g_3 & g_5 & -(\overline{\omega}^2-g_5) & 0 & 0 & 0 \\ 0 & 0 & 0 & \overline{\omega}^2-g_1 & g_2 & g_3 \\ 0 & 0 & 0 & g_2 & -(\overline{\omega}^2-g_4) & g_5 \\ 0 & 0 & 0 & g_3 & g_5 & -(\overline{\omega}^2-g_5) \end{bmatrix} \begin{bmatrix} U_1 \\ V_1 \\ W_1 \\ U_2 \\ V_2 \\ W_2 \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix} \tag{4.16}$$

Where,

$$\begin{aligned} g_1 &= \overline{\gamma}^2+\frac{(1-\nu)\overline{h}^2n^2}{2}, g_2=\frac{i(1+\nu)\overline{h}n\overline{\gamma}}{2}, \\ g_3 &= i\nu\overline{h}\overline{\gamma}, g_4=\overline{h}^2n^2+\frac{(1-\nu)\overline{\gamma}^2}{2}+(1-\nu^2)\overline{\eta}(\overline{h}^2n^2+\overline{\gamma}^2), \\ g_5 &= n\overline{h}^2, g_6=(1-\nu^2)\overline{\gamma}^2(\overline{N}-\overline{T(x)}-\overline{\eta})-\overline{h}^2-\varsigma \end{aligned}$$

A trivial solution is obtained by solving the matrix given in Eqn (4.16), so as to achieve a significant solution compare the coefficient of the determinant arrangement to nonexistent as follows

$$\begin{vmatrix} \overline{\omega}^2 - g_1 & g_2 & g_3 & 0 & 0 & 0 \\ g_2 & -(\overline{\omega}^2 - g_4) & g_5 & 0 & 0 & 0 \\ g_3 & g_5 & -(\overline{\omega}^2 - g_5) & 0 & 0 & 0 \\ 0 & 0 & 0 & \overline{\omega}^2 - g_1 & g_2 & g_3 \\ 0 & 0 & 0 & g_2 & -(\overline{\omega}^2 - g_4) & g_5 \\ 0 & 0 & 0 & g_3 & g_5 & -(\overline{\omega}^2 - g_5) \end{vmatrix} = 0 \tag{4.17}$$

By solving the determinant given in Eqn (4.17), a twelfth order dimensionless frequency equation is obtained in the form

$$\overline{\omega}^{12} - a_1\overline{\omega}^{10} + a_2\overline{\omega}^8 + a_3\overline{\omega}^6 - a_4\overline{\omega}^4 + a_5\overline{\omega}^2 - a_6 = 0 \tag{4.18}$$

where,

$$\begin{aligned} a_1 &= s_1 + g_4 + g_2 + g_6, \\ a_2 &= s_2 + s_1(g_4 + g_6) + s_1 g_1 + g_1(g_4 + g_6) + (g_4 g_6 - g_5^2) + g_2^2 - g_3^2, \\ a_3 &= s_1(g_4 + g_6) + s_3 - s_1(g_4 g_6 - g_5^2) - s_2 g_1 - s_1 g_1(g_4 + g_6) \\ &\quad - (g_1 g_4 g_6 - g_1 g_5^2)(s_1 g_2^2 + g_2(g_2 g_6 - g_3 g_5)) + s_1 g_3^2 + g_3(g_4 g_3 + g_5), \\ a_4 &= s_3(g_4 + g_6) + s_1 g_1(g_4 + g_6) + s_3 g - s_1 g_1(g_4 g_6 - g_5^2) \\ &\quad - (s_2 g_2^2 + s_1 g_2(g_2 g_6 - g_3 g_5)) + s_2 g_3^2 + s_1 g_3(g_4 g_3 + g_5), \\ a_5 &= s_2(g_4 g_6 - g_5^5) + s_3 g_1(g_4 + g_6) + s_3(g_4 g_6 - g_5^2) \\ &\quad + s_3 g_2^2 - s_2 g_2(g_2 g_6 - g_3 g_5) + s_2 g_3(g_4 g_3 + g_5) - s_3 g_3^2, \\ a_6 &= s_2 g_1(g_4 g_6 - g_5^2) + s_3 g_1(g_4 g_6 - g_5^2) + s_3 g_2(g_2 g_6 - g_3 g_5) - s_3(g_4 g_3^2 + g_5^2) \end{aligned}$$

in which,

$$\begin{aligned} s_1 &= g_4 + g_6 + g_1, \\ s_2 &= g_4 g_6 + g_1 g_4 - g_5^2 + g_1 g_6 + g_2^2 + g_3^2, \\ s_3 &= g_1 g_5^2 - g_1 g_4 g_6 \end{aligned}$$

By using the relation $\overline{\omega} = \overline{\gamma c}$ in Eqn (4.18) the dimensionless frequency equation can be changed in-terms of phase velocity as follows

$$\overline{c}^{12} - a_7\overline{c}^{10} + a_8\overline{c}^8 + a_9\overline{c}^6 - a_{10}\overline{c}^4 + a_{11}\overline{c}^2 - a_{12} = 0 \tag{4.19}$$

Where,

$$a_7 = \frac{1}{\gamma^2}[s_1 + g_4 + g_2 + g_6],$$

$$a_8 = \frac{1}{\gamma^4}[s_2 + s_1(g_4 + g_6) + s_1 g_1 + g_1(g_4 + g_6) + (g_4 g_6 - g_5^2) + g_2^2 - g_3^2],$$

$$a_9 = \frac{1}{\gamma^6}[s_1(g_4 + g_6) + s_3 - s_1(g_4 g_6 - g_5^2) - s_2 g_1 - s_1 g_1(g_4 + g_6) - (g_1 g_4 g_6 - g_1 g_5^2)$$
$$- (s_1 g_2^2 + g_2(g_2 g_6 - g_3 g_5)) + s_1 g_3^2 + g_3(g_4 g_3 + g_5)],$$

$$a_{10} = \frac{1}{\gamma^8}[s_3(g_4 + g_6) + s_1 g_1(g_4 + g_6) + s_3 g - s_1 g_1(g_4 g_6 - g_5^2)$$
$$- (s_2 g_2^2 + s_1 g_2(g_2 g_6 - g_3 g_5)) + s_2 g_3^2 + s_1 g_3(g_4 g_3 + g_5)],$$

$$a_{11} = \frac{1}{\gamma^{10}}[s_2(g_4 g_6 - g_5^5) + s_3 g_1(g_4 + g_6) + s_3(g_4 g_6 - g_5^2) + s_3 g_2^2$$
$$- s_2 g_2(g_2 g_6 - g_3 g_5) + s_2 g_3(g_4 g_3 + g_5) - s_3 g_3^2],$$

$$a_{12} = \frac{1}{\gamma^{12}}[s_2 g_1(g_4 g_6 - g_5^2) + s_3 g_1(g_4 g_6 - g_5^2) + s_3 g_2(g_2 g_6 - g_3 g_5) - s_3(g_4 g_3^2 + g_5^2)]$$

5 Numerical Results

In this paper, the scattered relations of double walled graphene tubule with the magnetic, thermal and rotating force is inferred utilizing shell theory. From Liew and Wang [15] the material boundaries are considered as in plane solidness $Eh = 360J/m^2$, mass thickness $h = 0.34nm$ and Poisson proportion $\nu = 0.3$, temperature $T = 200K$ and the consistent of thermal extension $\alpha_x = -1.6 \times 10^-6K^{-1}$. The scattering bends are examined for two modes of vibrations for $n = 0$ and $n = 1$. Diagrams are drawn between the dimensionless wavenumber and dimensionless

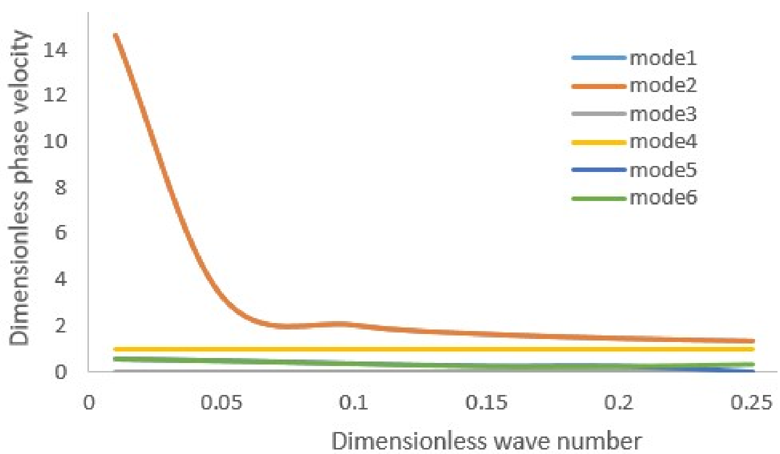

Figure 2. Scattered bends of double walled graphene tubule subjected to thermal, rotating and magnetic force of mode $n = 0$

phase velocity of graphene tubule under the thermal, magnetic and rotating forces of modes $n = 0$ and $n = 1$ and are appeared in Figures 2 and 3. From Figures 2 and 3, it is seen that proliferation of waves in the graphene tubule under the impact of thermal, magnetic and revolution force shows up in six modes. Also the non-dimensional phase velocity lessens as non-dimensional wavenumber risings for all methods of vibrations. Charts are drawn between the dimensionless wavenumber and dimensionless phase velocity of double walled graphene tubule exposed to thermal, rotating and magnetic fields at various magnetic field of mode $n = 0$ and $n =$

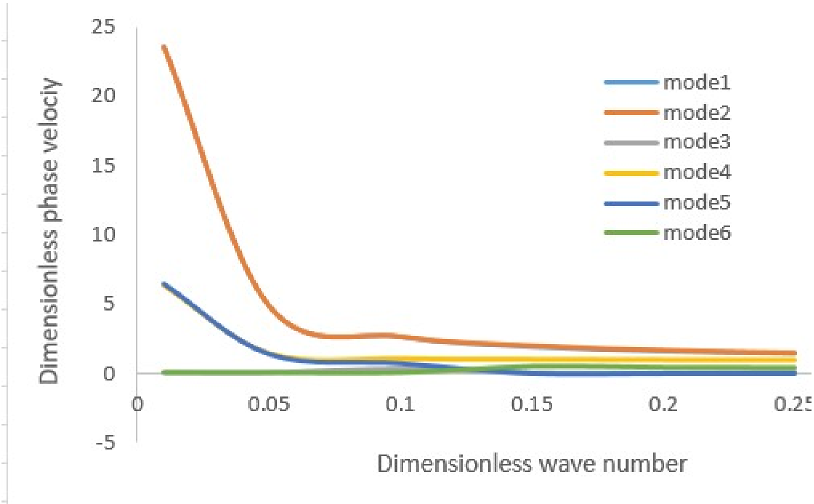

Figure 3. Scattered bends of double walled graphene tubule subjected to thermal, rotating and magnetic force of mode $n = 1$

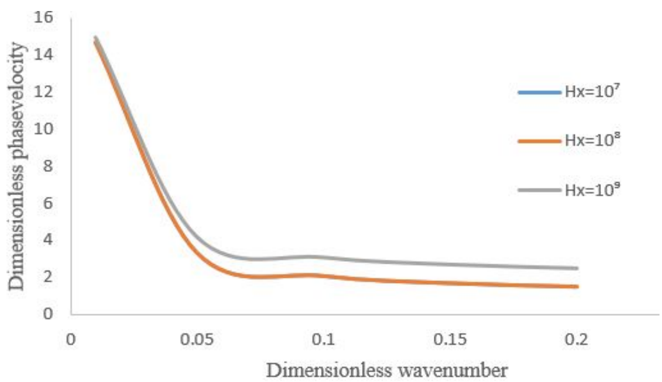

Figure 4. Scattered bends of double walled graphene tubule subjected to thermal, rotating and magnetic force for different magnetic field of mode $n = 0$.

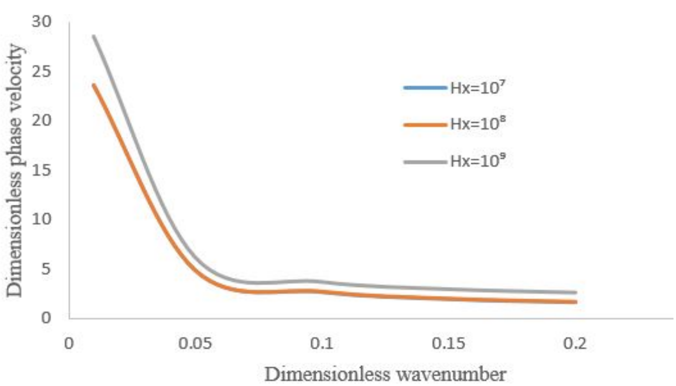

Figure 5. Scattered bends of double walled graphene tubule subjected to thermal, rotating and magnetic force for different magnetic field of mode $n = 1$.

1 and are appeared in Figures 4 and 5. From Figures 4 and 5 it is seen that fields of magnetic force increment the dimensionless phase velocity increments in all methods of vibrations. In addition, it is seen that the non-dimensional phase velocity diminishes as non-dimensional wave number ascent for any vibrational modes. Scattered bends of double walled graphene tubule under

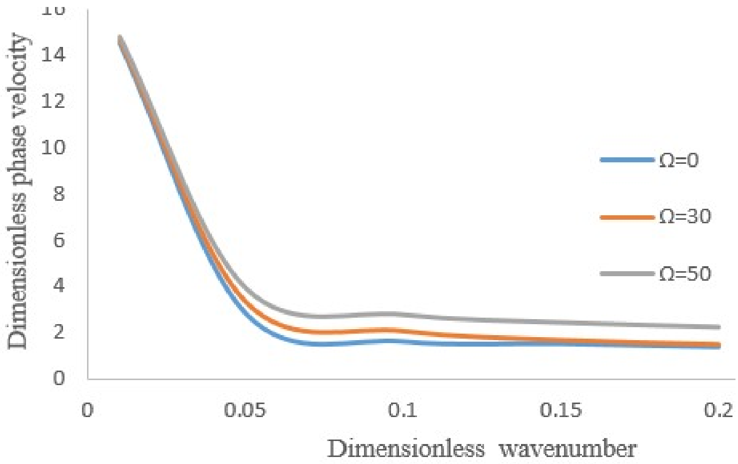

Figure 6. Scattered bends of double walled graphene tubule subjected to thermal, rotating and magnetic force for different rotating field of mode $n = 0$.

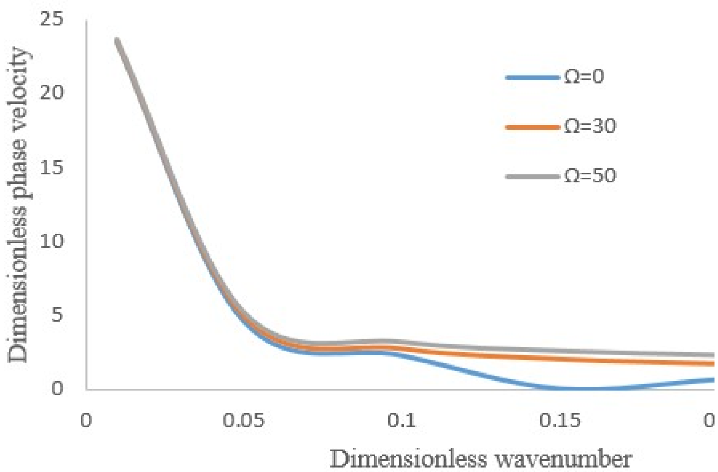

Figure 7. Scattered bends of double walled graphene tubule subjected to thermal, rotating and magnetic force for different rotating field of mode $n = 1$.

thermal, rotating and magnetic field at various rotating force for mode $n = 0$ and $n = 1$ are drawn and are appeared in Figures 6 and 7. From Figures 6 and 7, it is uncovered that the huge values of spinning speed its dimensionless wave number gets expanded. Moreover with expanding the rotating force, the dimensionless phase velocity increments with the dimensionless wave number for all modes of vibration. Scattered bends for double walled graphene tubule under magnetic, thermal and rotating force at various temperature for mode $n = 0$ and mode $n = 1$ are drawn and are appeared in Figures 8 and 9. From Figures 8 and 9, it is seen that temperature change consequentially affects dimensionless phase velocity. It is seen that dimensionless phase velocity decline with the ascent in temperature change. Besides, dimenensionless phase velocity is asymptotic to the dimensionless wavenumber. Scattered bends for single walled graphene tubule under magnetic, thermal and rotating force for mode $n = 0$ and mode $n = 1$ are drawn and are appeared in Figures 10 and 11. From Figures 10 and 11, it is seen that dimensionless

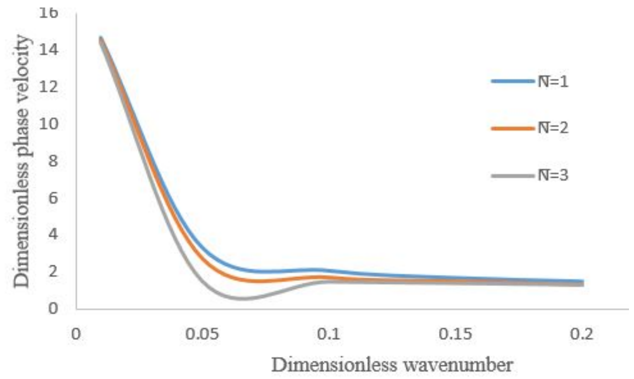

Figure 8. Scattered bends of double walled graphene tubule subjected to thermal, rotating and magnetic force for different thermal field of mode $n = 0$.

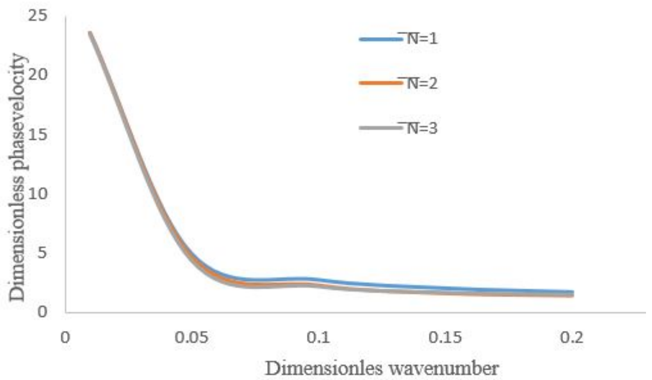

Figure 9. Scattered bends of double walled graphene tubule subjected to thermal, rotating and magnetic force for different thermal field of mode $n = 1$.

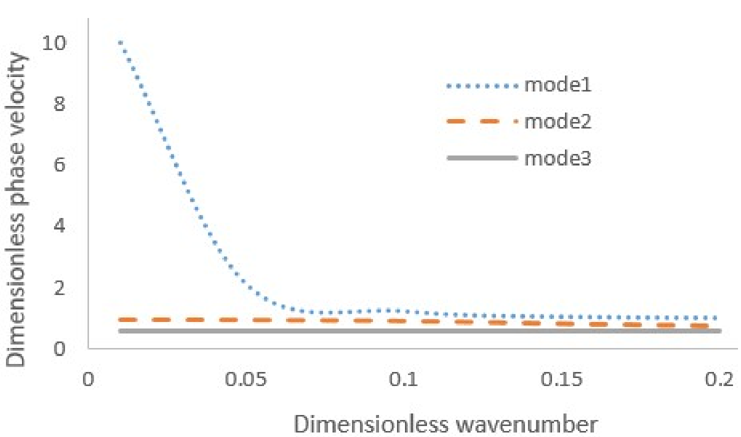

Figure 10. Scattered bends of single walled graphene tubule subjected to thermal, rotating and magnetic force for mode $n = 0$.

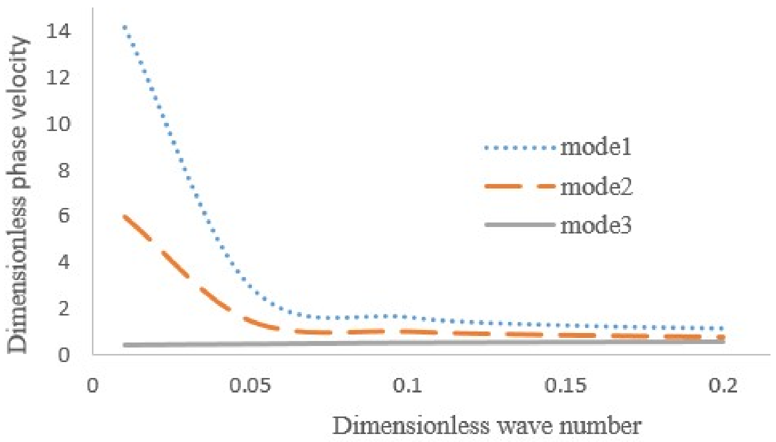

Figure 11. Scattered bends of single walled graphene tubule subjected to thermal, rotating and magnetic force for mode $n = 1$.

phase velocity decline with the ascent in dimensionless wavenumber for all modes of vibrations. Besides, dimenensionless phase velocity approaches to a certain value 0.5917. Scattered bends

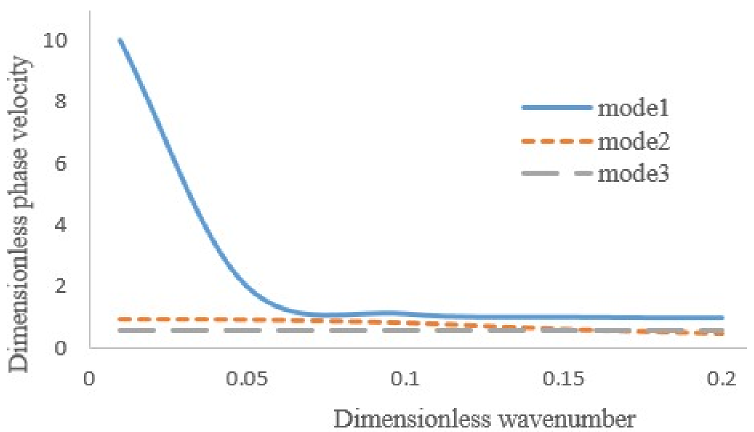

Figure 12. Scattered bends of single walled graphene tubule without the thermal, rotating and magnetic force for mode $n = 0$.

of the single walled graphene tubule in the void of magnetic, rotating and thermal force for mode $n = 0$ is drawn and is appeared in Figure 12. From Figure 12, it is seen that the wave engendering of single walled graphene tubule happens in three vibrational mode. The scattered bend of the graphene tubule in void of magnetic, rotating and thermal force matches with Figure 4.23 of Graff [16] and it displays the precision of the current paper.

6 Conclusion

In the current paper, the effect of thermo magnetic impact on the distribution of waves of double walled graphene tubule are considered utilizing shell theory. The scattered relations between dimensionless phase velocity and dimensionless wavenumber is derived by finding the solution in matrix form. Also with drives it is reasoned that the non-dimensional phase velocity reduces as dimensionless wave number climbs for any methods of vibration. Moreover, the dimensionless stage speeds increments as expansion in the magnetic and rotating force. Likewise the non-dimensional phase velocity decreases as expansion in temperature change.

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