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$\mathcal{A}$-Expansion $\beta^{\star}$g$\alpha(\mu,\lambda)$ -continuous in Hereditary generalized topological spaces

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Abstract In this paper, the concept of $\mathcal{A}$-Expansion $\beta^{\star}g\alpha(\mu,\lambda)$-continuity is introduced. Also some properties and characterization of $\mathcal{A}$-Expansion $\beta^{\star}g\alpha(\mu,\lambda)$-continuity are discussed briefly.

1 Introduction

The idea of generalized topology and hereditary class was introduced and studied by Csaszar[3, 4]. A subfamily μ of $P(X)$ is called a generalized topology if $\Phi\in\mu$ and union of elements of μ belongs to μ. And we say a hereditary class $\mathcal{H}$ on X is a non-empty collection of subset of X such that $A\subset B$, $B\in\mathcal{H}$ implies $A\in\mathcal{H}$. With respect to the generalised topology μ and a hereditary class $\mathcal{H}$, for a subset A of X we define $A^{\star}_{\mu}(\mathcal{H})$ [4] as $A^{\star}_{\mu}(\mathcal{H})$ = $\{x\in X\colon M\cap A\notin\mathcal{H}$ for every $M\in\mu$ such that $x\in M\}$. The closure $c^{\star}_{\mu}(A)=A\cup A^{\star}_{\mu}$. $\mathcal{H}$ is said to be μ-codense[4] if $\mu\cap\mathcal{H}=\{\phi\}$. With respect to μ we define $\mu^{\star}=\{A\subset X: c_{\mu^{\star}}(X-A)=X-A\}$. $i_{\mu^{\star}}(A)$ and $c_{\mu^{\star}}(A)$ will denote the interior and closure of A in $(X,\mu^{\star})$. The space (X,μ) with the hereditary class is called hereditary generalized topological space and denoted by $(X,\mu,\mathcal{H})$. A subset A of (X,μ) is $\mu\alpha$-open [3] (resp.μ-semi open [3], μ-pre open [[3]], μ-β-open[3]), if $A\subseteq i_{\mu}(c_{\mu}(i_{\mu}(A)))$ (resp.$A\subseteq(c_{\mu}(i_{\mu}(A))$,$A\subseteq i_{\mu}(c_{\mu}(A))$,$A\subseteq c_{\mu}(i_{\mu}(c_{\mu}(A)))$.We denote the family of all $\mu\alpha$-open sets, μ-semiopen sets, μ-pre open sets and $\mu\beta$-open sets by $\alpha(\mu)$, $\sigma(\mu)$, $\pi(\mu)$ and $\beta(\mu)$ respectively. On GT, $\mu\subseteq\alpha(\mu)\subseteq\pi(\mu)\subseteq\beta(\mu)$. In GTS $c_{\mu_{\alpha}}(A)$ and $c_{\mu_{\beta}}(A)$ denotes α-closure and β-closure respectively.
A subset A of (X,μ) is said to be μ regular open[5] if $A=i_{\mu}(c_{\mu}(A))$. The finite union of regular open set is called $\mu\pi$-open sets and its complement is $\mu\pi$-closed set. A set A is said to be μg-closed [3] if $c_{\mu}(A)\subseteq U$, whenever $A\subseteq U$ and U is μ-open and its complement is μg-open.

Definition 1.1. For a subset A of hereditary generalized topological space $(X,\mu,\mathcal{H})$,
i)$A^{\star}_{\mu_{\alpha}}(\mathcal{H})$ [4] = $\{x\in X\colon M\cap A\notin\mathcal{H}$ for every $M\in\alpha(\mu)$ such that $x\in M\}$.
ii)$A^{\star}_{\mu_{\beta}}(\mathcal{H})$ [4]= $\{x\in X\colon M\cap A\notin\mathcal{H}$ for every $M\in\beta(\mu)$ such that $x\in M\}$.

2 $\mathcal{A}-$ Expansion $\beta^{\star}$g$\alpha(\mu,\lambda)$-*continuous*

Definition 2.1. A subset A of $(X,\mu$ is said to be
i)$g\alpha\mu$-closed if $c_{\mu_{\alpha}}(A)\subseteq U$, whenever $A\subseteq U$ and U is $\mu\alpha$-open and its complement is $g\alpha\mu$-open.
ii)$^{\star}g\alpha\mu$-closed if $c_{\mu}(A)\subseteq U$, whenever $A\subseteq U$ and U is $\mu g\alpha$-open and its complement is $^{\star}g\alpha\mu$-open.
iii)$\beta^{\star}g\alpha\mu$-closed if $c_{\mu_{\beta}}(A)\subseteq U$, whenever $A\subseteq U$ and U is $^{\star}g\alpha\mu$-open and its complement is $\beta^{\star}g\alpha\mu$-open.

Definition 2.2. [1] Let$(X$,μ, $\mathcal{H})$ be a hereditary generalised topological space. A map $\mathcal{A}$: $\mu\longrightarrow 2^{X}$ is said to be an expansion on $(X$,μ, $\mathcal{H})$ if $U\subseteq\mathcal{A}(U)$ for each $U\in\mu$.

Definition 2.3. [1]Let$(X$,μ, $\mathcal{H})$ be a hereditary generalised topological space.A pair of expansions $\mathcal{A}$ and $\mathcal{B}$ is said to be mutually dual if $\mathcal{A}(U)\cap\mathcal{B}(U)=U$ for each $U\in\mu$.

Definition 2.4. [1]Let (X,μ) be a generalized topological space. Let$(Y,\lambda,\mathcal{H})$ be a hereditary generalized topological space. Let $\mathcal{A}$ be an expansion on Y. A map f:$(X,\mu) \longrightarrow (Y,\lambda,\mathcal{H})$ is said to be $\mathcal{A}$-$expansion\ continuous$ if $f^{-1}(V) \subseteq i_\mu(f^{-1}(\mathcal{A}(V))$ for each $V \in \lambda$.

Definition 2.5. A map $f : (X,\mu) \longrightarrow (Y,\lambda)$ is said to be $\beta^\star g\alpha(\mu,\lambda)$-continuous iff $U \in \lambda$ implies that $f^{-1}(U) \in \beta^\star g\alpha\mu.$

Definition 2.6. Let (X,μ) be a generalized topological space. Let$(Y,\lambda,\mathcal{H})$ be a hereditary generalized topological space. Let $\mathcal{A}$ be an expansion on Y. A map $f : (X,\mu) \longrightarrow (Y,\lambda,\ \mathcal{H})$ is said to be $\mathcal{A}$-expansion $\beta^\star g\alpha(\mu,\lambda)$-continuous if $f^{-1}(V) \subseteq \beta^\star g\alpha i_\mu(f^{-1}(\mathcal{A}(V))$ for each $V \in \lambda$.

Theorem 2.7. *Let (X,μ) be a quasi topological space and $(Y,\lambda,\mathcal{H})$ be a hereditary generalized topological space. Let $\mathcal{A}$ and $\mathcal{B}$ be two mutually dual expansions on $(Y,\lambda,\mathcal{H})$. Then a map $f : (X,\mu) \longrightarrow (Y,\lambda,\mathcal{H}))$ is $\beta^\star g\alpha(\mu,\lambda)$-continuous if and only if f is $\mathcal{A}$-expansion $\beta^\star g\alpha(\mu,\lambda)$-continuous and $\mathcal{B}$-expansion $\beta^\star g\alpha(\mu,\lambda)$-continuous.*

Proof. Necessity: Given that $\mathcal{A}$ and $\mathcal{B}$ are mutually dual expansions on (Y,$\mathcal{A}$ and $\mathcal{B}$). So $\mathcal{A}(V) \cap \mathcal{B}(V) = V$ for each $V \in \lambda$ and hence $f^{-1}(V) = f^{-1}(\mathcal{A}(V)) \cap f^{-1}(\mathcal{B}(V))$. Since f is $\beta^\star g\alpha$ (μ,λ)-continuous, $f^{-1}(V)$=$\beta^\star g\alpha i_\mu(f^{-1}(V))$.
So, $f^{-1}(V) = \beta^\star g\alpha i_\mu(f^{-1}(\mathcal{A}(V)) \cap \beta^\star g\alpha i_\mu(f^{-1}(\mathcal{B}(V))$. Thus $f^{-1}(V) \subseteq \beta^\star g\alpha i_\mu(f^{-1}(\mathcal{A}(V))$ and $f^{-1}(V) \subseteq \beta^\star g\alpha i_\mu(f^{-1}(\mathcal{B}(V))$ for every $V \in \lambda$. Hence f is $\mathcal{A}$-expansion $\beta^\star g\alpha(\mu,\lambda)$-continuous and $\mathcal{B}$-expansion $\beta^\star g\alpha(\mu,\lambda)$-continuous.
Sufficiency: Let $\mathcal{A}$ and $\mathcal{B}$ be two mutually dual expansions on $(Y,\lambda,\mathcal{H})$ and f be $\mathcal{A}$-expansion $\beta^\star g\alpha(\mu,\lambda)$-continuous and $\mathcal{B}$-expansion $\beta^\star g\alpha(\mu,\lambda)$-continuous. Then we have $f^{-1}(V) \subseteq \beta^\star g\alpha i_\mu(f^{-1}(\mathcal{A}(V))$ for each $V \in \lambda$ and $f^{-1}(V) \subseteq \beta^\star g\alpha i_\mu(f^{-1}(\mathcal{B}(V))$ for every $V \in \lambda$. Also we have $\mathcal{A}(V) \cap \mathcal{B}(V) = V$. Therefore, $f^{-1}(\mathcal{A}(V)) \cap f^{-1}(\mathcal{B}(V)) = f^{-1}(V)$. Hence $\beta^\star g\alpha i_\mu(f^{-1}(V)) = \beta^\star g\alpha i_\mu(f^{-1}\ (\mathcal{A}(V)) \cap \beta^\star g\alpha\, i_\mu(f^{-1}\ (\mathcal{B}(V)) \supseteq f^{-1}(V) \cap f^{-1}(V) = f^{-1}(V)$. So $\beta^\star g\alpha i_\mu(f^{-1}(V)) \supseteq f^{-1}(V)$. But $\beta^\star g\alpha i_\mu(f^{-1}(V)) \subseteq f^{-1}(V)$. Therefore $f^{-1}(V) = \beta^\star g\alpha i_\mu(f^{-1}(V))$. This implies that $f^{-1}(V) \in \beta^\star g\alpha\mu$ for each $V \in \lambda$. Therefore f is $\beta^\star g\alpha(\mu,\lambda)$-continuous.

Let Γ be the set of all expansion on generalised topological space (X,μ),a partial order " $<$ " can be defined by a relation $\mathcal{A}\ <\ \mathcal{B}$ if and only if $\mathcal{A}\ (V)\ < \mathcal{B}\ (V)$ for all $V \in \mu$. □

Theorem 2.8. *Let (X,μ) be a generalised topological space and $(Y,\lambda,\mathcal{H})$ be a hereditary generalized topological space. $\mathcal{A}$ be an expansion on $(Y,\lambda,\mathcal{H})$. Let f:$(X,\mu) \longrightarrow (Y,\lambda,\mathcal{H})$ be $\mathcal{A}$-$expansion$ $\beta^\star g\alpha(\mu,\lambda)$-continuous. Then f is $\mathcal{B}$-$expansion$ $\beta^\star g\alpha(\mu,\lambda)$-$continuous$ for any expansion $\mathcal{B}$ on $(Y,\lambda,\mathcal{H})$ such that $\mathcal{A} < \mathcal{B}$*

Proof. Given that f is $\mathcal{A}$-$expansion$ $\beta^\star g\alpha(\mu,\lambda)$-continuous and $\mathcal{A}\ <\ \mathcal{B}$. Then for each $V \in \lambda$, $\mathcal{A}\ (V) < \mathcal{B}\ (V)$ and $f^{-1}(V) \subseteq \beta^\star g\alpha i_\mu(f^{-1}(\mathcal{A}\ (V)))$ and hence $f^{-1}(V) \subseteq \beta^\star g\alpha i_\mu(f^{-1}(\mathcal{B}\ (V)))$. Therefore, f is $mathcalB$-expansion $\beta^\star g\alpha(\mu,\lambda)$-$continuous$ for any expansion $\mathcal{B}$ on (Y,λ,$\mathcal{H}$) such that $\mathcal{A}\ <\ \mathcal{B}$ □

Definition 2.9. [1] Let (X,μ) be a generalized topological space and $(Y,\lambda,\mathcal{H})$ be a hereditary generalized topological space. Let $\mathcal{B}$ be an expansion on $(Y,\lambda,\mathcal{H})$. A map $f : (X,\mu) \longrightarrow (Y,\lambda,\mathcal{H})$ is said to be closed $\mathcal{A}$- $expansion\ continuous$ if $f^{-1}((\mathcal{A}\ (V))^c)$ is μ-$closed$ in (X,μ) for each $V \in \lambda$.

Definition 2.10. Let (X,μ) be a generalized topological space and $(Y,\lambda,\mathcal{H})$ be a hereditary generalized topological space. Let $\mathcal{A}$ be an expansion on $(Y,\lambda,\mathcal{H})$. A map $f : (X,\mu) \longrightarrow (Y,\lambda,\mathcal{H})$ is said to be closed $\mathcal{A}$- expansion $\beta^\star g\alpha(\mu,\lambda)$-$continuous$ if $f^{-1}((\mathcal{A}\ (V))^c)$ is $\beta^\star g\alpha\mu$-$closed$ in (X,μ) for each $V \in \lambda$.

Definition 2.11. [1] An expansion $\mathcal{A}$ on $(X,\mu,\mathcal{H})$ is said to be $open$ if $\mathcal{A}(V) \in \mu$ for all $V \in \mu$.

Definition 2.12. [1] An expansion $\mathcal{A}$ on $(X,\mu,\mathcal{H})$is said to be $idempotent$ if $\mathcal{A}(\mathcal{A}\ (V)) = \mathcal{A}\ (V)$ for all $V \in \mu$

Theorem 2.13. *Let f:(X,μ)$\longrightarrow$ (Y,λ,$\mathcal{H}$), where $\mathcal{H}$ is λ codense and $\mathcal{A}$ be an open and idempotent on (Y,λ,$\mathcal{H}$). Then f is $\mathcal{A}$- expansion $\beta^\star g\alpha(\mu,\lambda)$-continuous if and only if f is closed $\mathcal{A}$-expansion $\beta^\star g\alpha(\mu,\lambda)$-continuous*

Proof. Necessary part: Let f be $\mathcal{A}$- expansion $\beta^\star g\alpha(\mu,\lambda)$-continuous, where $\mathcal{A}$ is idempotent and $V \in \lambda$. Since $\mathcal{A}$ is open and idempotent, $\mathcal{A}\ (V) \in \lambda$ and $\mathcal{A}(\mathcal{A}\ (V))$ = $\mathcal{A}\ (V)$. Then $f^{-1}(\mathcal{A}\ (V)) \subseteq \beta^\star g\alpha i_\mu(f^{-1}(\mathcal{A}(\mathcal{A}\ (V))))$ = $\beta^\star g\alpha i_\mu(f^{-1}(\mathcal{A}\ (V)))$

Sufficient part:We prove $(f^{-1}(\mathcal{A}\ (V))^c)^c$ = $f^{-1}(\mathcal{A}\ (V))$. Let $x \in (f^{-1}(\mathcal{A}\ (V))^c)^c$. Then $x \notin f^{-1}(\mathcal{A}\ (V))^c)$ and $f(x) \notin (A\ (V))^c$. Hence $x \in f^{-1}(\mathcal{A}\ (V))$. So, $(f^{-1}(\mathcal{A}\ (V))^c)^c \subseteq f^{-1}(\mathcal{A}\ (V))$. Conversely, let $x \in f^{-1}(\mathcal{A}\ (V))$. Then $f(x) \in \mathcal{A}\ (V)$ and so $f(x) \notin (\mathcal{A}\ (V))^c$ which implies $x \notin f^{-1}((\mathcal{A}\ (V))^c)$ and $x \in (f^{-1}(\mathcal{A}\ (V))^c)^c$. So,$(f^{-1}(\mathcal{A}\ (V))^c)^c \subseteq f^{-1}(\mathcal{A}\ (V))$. Therefore, $(f^{-1}(\mathcal{A}\ (V))^c)^c$ = $f^{-1}(\mathcal{A}\ (V))$. Since f is *closed* $\mathcal{A}$- expansion $\beta^\star g\alpha(\mu,\lambda)$-*continuous*, $f^{-1}(\mathcal{A}\ (V))^c$ is $\beta^\star g\alpha\mu$-*closed* in (X,μ). This implies $(f^{-1}(\mathcal{A}\ (V))^c)^c \in \beta^\star g\alpha\mu$. Hence $f^{-1}(\mathcal{A}\ (V)) \in \beta^\star g\alpha\mu$ and so $f^{-1}(\mathcal{A}\ (V))$ = $\beta^\star g\alpha i_\mu(f^{-1}(\mathcal{A}\ (V)))$. Also $V \subseteq \mathcal{A}\ (V)$, and this implies $f^{-1}(V) \subseteq f^{-1}(\mathcal{A}\ (V))$ = $\beta^\star g\alpha i_\mu(f^{-1}(\mathcal{A}\ (V)))$. Therefore $f^{-1}(V) \subseteq \beta^\star g\alpha i_\mu(f^{-1}(\mathcal{A}\ (V)))$ for each $V \in \lambda$. Hence f is $\mathcal{A}$- expansion $\beta^\star g\alpha(\mu,\lambda)$-*continuous*. □

Definition 2.14. Let $\mathcal{A}$ be an expansion on a hereditary generalized topological space $(X,\mu,\mathcal{H})$, $\mathcal{A}$ and $\mathcal{B}$ be expansions on (X,μ) and $(Y,\lambda,\mathcal{H})$ respectively.We say that the map f:$(X,\ \mu) \longrightarrow (Y,\lambda,\mathcal{H})$ is $(\mathcal{A},\mathcal{B})$ *weakly*-continuous if and only if $\mathcal{A}\ (f^{-1}(V)) \subseteq i_\mu(f^{-1}(\mathcal{B}\ (V)))$ for every $V \in \lambda$.

Definition 2.15. Let $\mathcal{A}$ be an expansion on hereditary generalized topological space$(X,\mu,\mathcal{H})$, $\mathcal{A}$ and $\mathcal{B}$ be expansions on (X,μ) and $(Y,\lambda,\mathcal{H})$ respectively. We say that the map f:$(X,\ \mu) \longrightarrow (Y,\lambda,\mathcal{H})$ is $(\mathcal{A},\mathcal{B})$ *weakly* $\beta^\star g\alpha$-*continuous* if and only if $\mathcal{A}(f^{-1}(V)) \subseteq \beta^\star g\alpha i_\mu(f^{-1}(\mathcal{B}\ (V)))$ for every $V \in \lambda$.
The expansion $\mathcal{A}\ ((V))$ = V is called the identity expansion and it is denoted by $\mathcal{I}d$-*expansion*.

Definition 2.16. Let $(X,\mu,\mathcal{H})$ be a hereditary generalized topological space. An expansion $\mathcal{A}$ on $(X,\mu,\mathcal{H})$ is said to be *subadditive* if for every collection of μ- *open set* $\{U_\alpha : \alpha\in\triangle\}$, $\mathcal{A}(U_{\alpha\in\triangle}U_\alpha) \subseteq U_{\alpha\in\triangle}\mathcal{A}(U_\alpha)$.

Definition 2.17. Let $(X,\mu,\mathcal{H})$ be a hereditary generalised topological space. An expansion $\mathcal{A}$ on $(X,\mu,\mathcal{H})$ is said to be $\beta^\star g\alpha$-*subadditive* if for every collection of $\beta^\star g\alpha\mu$- *open set* $\{U_\alpha : \alpha\in\triangle\}$, $\mathcal{A}(U_{\alpha\in\triangle}U_\alpha) \subseteq U_{\alpha\in\triangle}\mathcal{A}(U_\alpha)$

Theorem 2.18. *Let (X,μ) be a quasi topological space and $(Y,\lambda,\mathcal{H})$ be a hereditary generalized topological space and $\mathcal{A}$ be an expansion on (X,μ). If $\mathcal{B}$ and $\mathcal{B}^\star$ are two mutually dual expansions on (Y,λ,$\mathcal{H}$) then a map f : (X,μ)$\longrightarrow$ (Y,λ,$\mathcal{H}$) is ($\mathcal{A}$ $\mathcal{I}d$) weakly $\beta^\star g\alpha$-continuous if and only if f is both ($\mathcal{A}$,$\mathcal{B}$) weakly $\beta^\star g\alpha$-continuous and ($\mathcal{A}$,$\mathcal{B}^\star$) weakly $\beta^\star g\alpha$-continuous*

Proof. Suppose f is $(\mathcal{A},\ \mathcal{I}d)$ *weakly* $\beta^\star g\alpha$-*continuous*. Then for every $V \in \lambda$, we have $\mathcal{A}\ (f^{-1}(V))\subseteq \beta^\star g\alpha\ i_\mu(f^{-1}(V)) \subseteq \beta^\star g\alpha i_\mu\ (f^{-1}(\mathcal{B}(V)))\cap\beta^\star g\alpha i_\mu(f^{-1}(\mathcal{B}^\star(V)))$. This implies that f is $(\mathcal{A},\mathcal{B})$ *weakly* $\beta^\star g\alpha$-*continuous* and $(\mathcal{A},\mathcal{B}^\star)$ weakly $\beta^\star g\alpha$-*continuous*. Conversely, suppose f is $(\mathcal{A},\mathcal{B})$weakly $\beta^\star g\alpha$-*continuous* and $(\mathcal{A},\mathcal{B}^\star)$ weakly $\beta^\star g\alpha$-*continuous*. Then for every $V \in \lambda$, we have $\mathcal{A}\ (f^{-1}(V)) \subseteq \beta^\star g\alpha i_\mu(f^{-1}(\mathcal{B}(V)))$ and $\mathcal{A}\ (f^{-1}(V)) \subseteq \beta^\star g\alpha i_\mu(f^{-1}(\mathcal{B}^\star(V)))$.
Thus $\mathcal{A}\ (f^{-1}(V)) \subseteq \beta^\star g\alpha i_\mu(f^{-1}(V)) \subseteq \beta^\star g\alpha i_\mu(f^{-1}(\mathcal{B}(V))) \cap \beta^\star g\alpha i_\mu(f^{-1}\ (\mathcal{B}^\star(V)))$ =
$\beta^\star g\alpha i_\mu(f^{-1}(\mathcal{B}(V)) \cap \mathcal{B}^\star(V)))$. Since $\mathcal{B}$ and $\mathcal{B}^\star$ are mutually dual expansions on (Y,λ,$\mathcal{H}$), we get $\mathcal{A}(f^{-1}(V)) \subseteq \beta^\star g\alpha i_\mu\ (f^{-1}(V))$, which implies that f is $(\mathcal{A},\ \mathcal{I}d)$ *weakly* $\beta^\star g\alpha$-*continuous*. □

Definition 2.19. Let $\mathcal{A}$ be an expansion on $(X,\ \mu,\mathcal{H})$. A subset A of X is said to be $\mathcal{A}$-*open* if for each $x \in A$ there exist $U \in \mu(x)$ such that $\mathcal{A}\ (U) \subseteq A$ and $\mathcal{A}$-*closed* if $X - A$ is $\mathcal{A}$-*open*.

Definition 2.20. Let $\mathcal{A}$ be an expansion on $(X,\ \mu,\ \mathcal{H})$. A subset A of X is said to be $\beta^\star g\alpha\mu$ $\mathcal{A}$-*open* if for each $x \in A$ there exist $U \in \beta^\star g\alpha\mu(x)$ such that $\mathcal{A}\ (U) \subseteq A$ and $\beta^\star g\alpha\mu$ $\mathcal{A}$-*closed* if $X - A$ is $\beta^\star g\alpha\mu$ $\mathcal{A}$-*open*.

Theorem 2.21. *Let f:$(X, \mu) \longrightarrow (Y,\lambda,\mathcal{H})$, where $\mathcal{H}$ is λ-$codense$ and $\mathcal{B}$ be an $open$ and $idempotent$ expansion on $(Y,\lambda,\mathcal{H})$ and if $\mathcal{A}$ is $subadditive$ and $monotone$ expansion on (X, μ). Then f is($\mathcal{A}$,$\mathcal{B}$)-$weakly$ $\beta^\star g\alpha$-$continuous$ if and only if closed ($\mathcal{A}$,$\mathcal{B}$)$\beta^\star g\alpha$-$continuous$.*

Proof. Necessary part: Let f be ($\mathcal{A}$,$\mathcal{B}$)-weakly $\beta^\star g\alpha$-$continuous$, where $\mathcal{B}$ is $open$ and $idempotent$, then $\mathcal{A}(f^{-1}(V)) \subseteq \beta^\star g\alpha i_\mu(f^{-1}$ $(\mathcal{B}(V)))$ for each $V\in\lambda$. Since $\mathcal{B}$ is $open$ and $idempotent$, $\mathcal{B}(V)$ $\in \lambda$ and $\mathcal{B}(\mathcal{B}(V)) = \mathcal{B}(V)$. Then $f^{-1}(\mathcal{B}(V)) \subseteq \mathcal{A}(f^{-1}(\mathcal{B}(V))) \subseteq \beta^\star g\alpha i_\mu(f^{-1}(\mathcal{B}(\mathcal{B}(V))))$=$\beta^\star g\alpha$ $i_\mu(f^{-1}(\mathcal{B}(V)))$. Thus $f^{-1}(\mathcal{B}(V)) \in \beta^\star g\alpha\mu$ and $\mathcal{A}(f^{-1}(\mathcal{B}(V)))$ = $f^{-1}(\mathcal{B}(V))$ and hence $(f^{-1}(\mathcal{B}(V)))^c$ = $f^{-1}((\mathcal{B}(V))^c)$ is $\beta^\star g\alpha\mu$ $\mathcal{A}$-$closed$. Therefore f is $closed$ ($\mathcal{A}$,$\mathcal{B}$)$\beta^\star g\alpha$-$continuous$.

Sufficient part: Let f:$(X, \mu) \longrightarrow (Y,\lambda,\mathcal{H})$ be $closed$ ($\mathcal{A}$,$\mathcal{B}$)$\beta^\star g\alpha$-continuous and let $V \in \lambda$.We prove $(f^{-1}(\mathcal{B}\,(V))^c)^c$=$f^{-1}(\mathcal{B}\,(V))$. Let x∈$(f^{-1}(\mathcal{B}(V))^c)^c$. Then $x\notin f^{-1}(\mathcal{B}(V))^c)$ and $f(x)\notin(\mathcal{B}(V))^c$. Hence $x \in f^{-1}(\mathcal{B}(V))$. So, $(f^{-1}(\mathcal{B}\,(V))^c)^c \subseteq f^{-1}(\mathcal{B}\,(V))$. Conversely, let $x \in f^{-1}(\mathcal{B}(V))$. Then $f(x) \in \mathcal{B}\,(V)$ and so $f(x) \notin (\mathcal{B}\,(V))^c$ which implies $x \notin f^{-1}((\mathcal{B}\,(V))^c)$ and $x \in (f^{-1}(\mathcal{B}(V))^c)^c$. So,$(f^{-1}(\mathcal{A}\,(V))^c)^c \subseteq f^{-1}(\mathcal{B}\,(V))$. Therefore, $(f^{-1}(\mathcal{B}\,(V))^c)^c$=$f^{-1}(\mathcal{B}\,(V))$. Since f is $closed$ ($\mathcal{A}$,$\mathcal{B}$)-$\beta^\star g\alpha$-$continuous$, $f^{-1}(\mathcal{B}\,(V))^c$ is $\beta^\star g\alpha\mu$ $\mathcal{A}$-$closed$ in (X,μ). This implies $(f^{-1}(\mathcal{B}\,(V))^c)^c \in$ $\beta^\star g\alpha\mu$. Hence $f^{-1}(\mathcal{B}\,(V)) \in \beta^\star g\alpha\mu$ and so $f^{-1}(\mathcal{B}\,(V))$=$\beta^\star g\alpha i_\mu(f^{-1}(\mathcal{B}\,(V)))$. Also $V \subseteq \mathcal{B}(V)$, and this implies $f^{-1}(V) \subseteq f^{-1}(\mathcal{B}(V))$=$\beta^\star g\alpha\; i_\mu(f^{-1}(\mathcal{B}\,(V)))$. Therefore $\mathcal{A}(f^{-1}(V)) \subseteq$ $\beta^\star g\alpha i_\mu(f^{-1}(\mathcal{B}\,(V)))$ for each $V \in \lambda$. Hence f is ($\mathcal{A}$,$\mathcal{B}$)-$weakly$ $\beta^\star g\alpha$-continuous. □

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