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# On $r$-Dynamic coloring of $R$-vertex corona of graphs

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**Abstract** In this paper, the results on $r$-dynamic coloring of $R$-vertex corona of graphs are extracted. i.e., $R$-vertex corona of path with cycle, path with complete graph, cycle with path and complete graph with path.

## 1 Introduction

The $R$- graphs concept was first introduced by D. M. Cvetkovic et. al.[3]. $R(G)$ is obtained by adding a vertices $w_e$ and joining $w_e$ to the vertices of $e$ where $e \epsilon E(G)$. The newly added vertices are denoted as $I(G)$ such that $I(G) = V(R(G))/V(G)$. It is can also named as an edge corona, which is from $R(G)$ to a singleton graph. The concept $R(G)$ are widely in the spectra of graph theory which deals with the spectra of matrices that associates with graph and the properties of graph. Several researchers have also examined $R$-graph with many operations such as $R$-vertex corona, $R$-edge corona, the $R$-vertex neighbourhood corona, the $R$-edge neighbourhood corona and so on. This can be seen in the following papers [2], [4], [6]. Now, in this article we have combined the $R$-vertex corona with $r$- dynamic coloring.

The $r$-dynamic coloring which was introduced by Montgomery [8]. It is an proper vertex coloring which is extended from dynamic coloring. An $r$-dynamic coloring is defined by the following mapping $c : V(G) \rightarrow \{1, 2, \cdots, k\}$ it satisfy two condition:

$$c(u) \neq c(v) \tag{1.1}$$

$$|c(N(v)| \geqslant min\,\{r, d(v)\} \tag{1.2}$$

Where, $N(v)$ denote the neighborhood vertices of $v$ and $d(v)$ denote the degree of the vertex $v$. At $r = 1$ the 1-dynamic chromatic number is equal to the chromatic number of the graph and at $r = 2$ it is called as dynamic chromatic number. The $r$-values can be extended upto to the maximum degree $\Delta$. Montogomery have also shows that $\chi_2(G) - \chi(G) \leq 2$, for $r$-regular graph. Also, the bounds of $r$-dynamic coloring was given minimum and maximum degree. The $r$-dynamic chromatic number of some graphs and their bounds are studied in [1], [5], [7], [9].

In the next section, we study the basic prelimiaries of graph theory and some preliminary lemmas which can be used in the section 3. In the section 3 the exact values of $r$- dynamic coloring of $R$-vertex corona of graphs such as path with cycle, path with complete graph, cycle with path and complete graph with path are given.

## 2 Preliminaries

In this section, we deals with some basic definitions and some preliminary lemmas which are carried throughout the next section. A graph $G$ consist of a pair $(V(G), E(G))$ where $V(G)$ is the set of vertices and $E(G)$ are the edges. A graph which has an identical ends at the edge are called as *loop*. A graph which has no loops, and undirected graph are said to be *simple*. Then a graph is said to be *finite* if the order and size of the graph are finite. In this work, we considered the simple, connected and undirected graph. The maximum and minimum degrees of the graph $G$ are denoted as $\delta(G)$ and $\Delta(G)$.

A *path* between two distinct vertices $v$ and $w$ of $G$ are in sequence of ordered adjacent and $(q_0 = v, q_1, q_2, \cdots, q_{l-1} = w)$ in $V(G)$ are pairwise adjacent. A *cycle* is with $n$ vertices $(n \geq 3)$

and $n$ edges. It is a sequence of the vertices that begins and end at the same vertex, so that each vertex is with degree 2. A graph is said to be *Complete* if any of the two vertices are adjacent. It is denoted as $K_n$.

An proper $r$-dynamic coloring is a map of $c : V(G) \to \{1, 2, \cdots, k\}$ which assigns $k$-colors to the vertices. An $r$-dynamic chromatic number is the minimal coloring of a graph $G$ which is $r$-dynamic $k$-colorable.

$R$-vertex corona of $G_1$ and $G_2$ are constructed from the vertex disjoint $R(G_1)$ and $|V(G_1)|$ copies of $G_2$ by combining the $j$-th vertex of $G_1$ to every vertex in the $j$-th copy of $G_2$. It is denoted as $G_1^{(R)} \odot G_2$. The following lemmas and theorem are the basic results which are needed for the next section.

**Lemma 2.1.** *Let G be an finite, connected graph. the following conditions hold:*
*(1)* $\chi_r(G) \leq \chi_{r+1}(G)$.
*(2)* $\chi_r(G) \geq min\{r, \Delta(G)\} + 1$.
*(3)* $\chi(G) = \chi_1(G) \leq \chi_2(G) \leq \cdots \leq \chi_{\Delta(G)}(G)$.
*(4) At* $r \geq \Delta(G)$, *then* $\chi_r(G) = \chi_{\Delta(G)}(G)$.

**Theorem 2.2.** *For any positive integer* $n \geq 1$, *then* $\chi_r[(K_n)] = n$.

**Theorem 2.3.** *For any positive integers* $r \geq 2$ *and* $n$, *then*

$$\chi_r[(C_n)] = \begin{cases} 5, & for\ n = 5 \\ 3, & for\ n = 0(\text{mod}3) \\ 4, & otherwise \end{cases} \tag{1.3}$$

In the next section, the $r$-dynamic coloring deals with $R$-vertex corona of some connected graphs such as path with cycle, path with complete graph, cycle with path and complete graph with path.

## 3 Main Results

**Theorem 3.1.** *For any positive integers* $r$, $m \geq 2$ *and* $n \geq 3$, *the* $r$- *dynamic coloring of* $R$-*vertex corona of path with cycle* $P_m^{(R)} \odot C_n$ *are*

$$\chi_r[P_m^{(R)} \odot C_n] = \begin{cases} 3, & for\ 1 \leq r \leq 2,\ n\ is\ even \\ 4, & for\ 1 \leq r \leq 2,\ n\ is\ odd \\ 4, & for\ r = 3,\ n \equiv 0(\text{mod } 3) \\ 5, & for\ 3 \leq r \leq 4,\ n \equiv 1, 2(\text{mod } 3) \\ 6, & for\ 3 \leq r \leq 4,\ n = 20l + 5,\ l \in W \\ r + 1, & otherwise \end{cases}$$

**Proof.** Let $V[P_m^{(R)} \odot C_n] = \{u_i, u'_k, u_{ij} : i\epsilon[1, m], j\epsilon[1, n], k\epsilon[1, m-1]\}$, where $u_i$ be the vertices of path graph $P_m$, vertices of $u'_k$ be the vertices of $R$-graph of path $P_m^{(R)}$ and $u_{ij}$ be the vertices of $C_n$. In $R$-vertex corona of $P_m^{(R)} \odot C_n$ the m-copies of $C_n$ are connected to each vertices of $P_m$. The order and the size of the graph $P_m^{(R)} \odot C_n$ are $|V[P_m^{(R)} \odot C_n]| = mn + (2m - 1)$ and $|E[P_m^{(R)} \odot C_n]| = 3(m-1)+m(2n)$. The minimum and maximum degree are $\delta(P_m^{(R)} \odot C_n) = 2$, $\Delta(P_m^{(R)} \odot C_n) = n + 2$ for $m = 2$ and $\Delta(P_m^{(R)} \odot C_n) = n + 4$ for $m \geq 2$.

**Case :1** $1 \leq r \leq 2$

To determine $1, 2$-dynamic coloring of $P_m^{(R)} \odot C_n$, consider a function $c_1$, such that $c_1 : V[P_m^{(R)} \odot C_n] \to \{1, 2, \cdots, k\}$

$$\begin{aligned} c_1(u_i) &= \{1, 2\},\ for\ 1 \leq i \leq m \\ c_1(u'_k) &= 3,\ for\ 1 \leq k \leq m - 1 \end{aligned}$$

When $n$ is even,

$$\begin{aligned} c_1(u_{ij}) &= \{2,3\},\ for\ i\ is\ odd,\ 1 \leq i \leq m,\ 1 \leq j \leq n \\ c_1(u_{ij}) &= \{1,3\},\ for\ i\ is\ even,\ 1 \leq i \leq m,\ 1 \leq j \leq n \end{aligned}$$

Thus, $\chi_r[P_m^{(R)} \odot C_n] \leq 3$. From the lemma 2.1, it is clear that $\chi_r[P_m^{(R)} \odot C_n] \geq 3$. Hence, it is easy to prove that $\chi_r[P_m^{(R)} \odot C_n] = 3$.
When $n$ is odd,

$$\begin{aligned} c_1(u_{ij}) &= \{2,3\}\ for\ i\ is\ odd,\ 1 \leq i \leq m,\ 1 \leq j \leq n \\ c_1(u_{ij}) &= \{1,3\}\ for\ i\ is\ even,\ 1 \leq i \leq m,\ 1 \leq j \leq n-1 \\ c_1(u_{in}) &= 4\ for\ 1 \leq i \leq m \end{aligned}$$

Thus, $\chi_r[P_m^{(R)} \odot C_n] \leq 4$. From the lemma 2.1, it is clear that $\chi_r[P_m^{(R)} \odot C_n] \geq 4$. Hence, it is easy to prove that $\chi_r[P_m^{(R)} \odot C_n] = 4$.

**Case :2** $r = 3,\ n \equiv 0(\text{mod } 3)$

To determine the 3-dynamic coloring of $P_m^{(R)} \odot C_n$, we must show that $\chi_3[P_m^{(R)} \odot C_n] \geq 4$ and $\chi_3[P_m^{(R)} \odot C_n] \leq 4$. To prove the upper bound $\chi_4[P_m^{(R)} \odot C_n] \leq 4$, consider a function $c_2$, such that $c_2 : V[P_m^{(R)} \odot C_n] \to \{1, 2, \cdots, k\}$.

$$\begin{aligned} c_2(u_i) &= \{1,2\},\ for\ 1 \leq i \leq m \\ c_2(u'_k) &= 3,\ for\ 1 \leq k \leq m-1 \\ c_2(u_{ij}) &= \{2,3,4\}\ for\ 1 \leq i \leq m,\ 1 \leq j \leq n \end{aligned}$$

Thus, $\chi_r[P_m^{(R)} \odot C_n] \leq 4$. From the lemma 2.1, it is clear that $\chi_r[P_m^{(R)} \odot C_n] \geq 4$. Hence, it is easy to prove that $\chi_r[P_m^{(R)} \odot C_n] = 4$.

**Case :3** $3 \leq r \leq 4$

Consider a function $c_3$ such that $c_3 : V[P_m^{(R)} \odot C_n] \to \{1, 2, \cdots, k\}$.

- When $n \equiv 1, 2(\text{ mod } 3)$, we must show that $\chi_r[P_m^{(R)} \odot C_n] \geq 5$ and $\chi_r[P_m^{(R)} \odot C_n] \leq 5$. To prove, $\chi_r[P_m^{(R)} \odot C_n] \leq 5$, consider the following coloring:

$$\begin{aligned} c_3(u_i) &= \{1,2\},\ for\ 1 \leq i \leq m \\ c_3(u'_k) &= 3,\ for\ 1 \leq k \leq m-1 \\ c_3(u_{ij}) &= \{2,3,4,5\}\ for\ 1 \leq i \leq m,\ 1 \leq j \leq n \end{aligned}$$

  Thus, $\chi_r[P_m^{(R)} \odot C_n] \leq 5$. From the lemma 2.1, it is clear that $\chi_r[P_m^{(R)} \odot C_n] \geq 5$. Hence, it is easy to prove that $\chi_r[P_m^{(R)} \odot C_n] = 5$.

- When $n = 20l + 5,\ l \in W$, we must show that $\chi_r[P_m^{(R)} \odot C_n] = 6$. To prove this condition, we must prove $\chi_r[P_m^{(R)} \odot C_n] \geq 6$ and $\chi_r[P_m^{(R)} \odot C_n] \leq 6$. To prove, $\chi_r[P_m^{(R)} \odot C_n] \leq 6$, consider the below coloring:

$$\begin{aligned} c_3(u_i) &= \{1,2\},\ for\ 1 \leq i \leq m \\ c_3(u'_k) &= 3,\ for\ 1 \leq k \leq m-1 \\ c_3(u_{ij}) &= \{2,3,4,5,6\}\ for\ i\ is\ odd,\ 1 \leq i \leq m,\ 1 \leq j \leq n \\ c_3(u_{ij}) &= \{1,3,4,5,6\}\ for\ i\ is\ even,\ 1 \leq i \leq m,\ 1 \leq j \leq n \end{aligned}$$

Thus, $\chi_r[P_m^{(R)} \odot C_n] \leq 6$. From the lemma 2.1, it is proved that $\chi_r[P_m^{(R)} \odot C_n] \geq 6$. Therefore, it is easy to show that $\chi_r[P_m^{(R)} \odot C_n] = 6$.

**Case :4** *otherwise*
To determine the $r$-dynamic coloring of $P_m^{(R)} \odot C_n$, we must show that $\chi_r[P_m^{(R)} \odot C_n] \geq r+1$ and $\chi_r[P_m^{(R)} \odot C_n] \leq r+1$. To prove the upper bound $\chi_r[P_m^{(R)} \odot C_n] \leq r+1$, consider a function $c_4$, such that $c_4 : V[P_m^{(R)} \odot C_n] \rightarrow \{1, 2, \cdots, k\}$.

- When $5 \leq r \leq \Delta - 1$, consider the below coloring:
  - ∗ For $5 \leq r \leq \Delta - 2$, the coloring of the vertices $P_m^{(R)} \odot C_n$ are as follows,

$$\begin{aligned} c_4(u_i) &= \{1, 2\},\ for\ 1 \leq i \leq m \\ c_4(u'_k) &= 3,\ for\ 1 \leq k \leq m-1 \end{aligned}$$

    The coloring of the vertices $u_{ij}$ depends on the $r$-value, so we may color the vertices $u_{ij}$ from the set of colors $\{4, 5, \cdots, r+1\}$ for $1 \leq i \leq m$ and $1 \leq j \leq n$.
  - ∗ For $r = \Delta - 1$, the coloring are,

$$\begin{aligned} c_4(u_i) &= \{1, 3\},\ for\ 1 \leq i \leq m \\ c_4(u'_k) &= \{2, 4\},\ for\ 1 \leq k \leq m-1 \\ c_4(u_{ij}) &= \{5, 6, \cdots, r+1\}\ for\ 1 \leq i \leq m,\ 1 \leq j \leq n \end{aligned}$$

  Thus, $\chi_r[P_m^{(R)} \odot C_n] \leq r+1$. From the lemma 2.1, it is proved that $\chi_r[P_m^{(R)} \odot C_n] \geq r+1$. Hence, we can show that $\chi_r[P_m^{(R)} \odot C_n] = r+1$.
- When $r = \Delta$, the coloring of the vertices $P_m^{(R)} \odot C_n$ are as follows,

$$\begin{aligned} c_4(u_i) &= \{1, 3, 5\},\ for\ 1 \leq i \leq m \\ c_4(u'_k) &= \{2, 4\},\ for\ 1 \leq k \leq m-1 \\ c_4(u_{ij}) &= \{6, 7, \cdots, r+1\}\ for\ 1 \leq i \leq m,\ 1 \leq j \leq n \end{aligned}$$

Thus, $\chi_r[P_m^{(R)} \odot C_n] \leq r+1$. From the lemma 2.1, it is proved that $\chi_r[P_m^{(R)} \odot C_n] \geq r+1$. Therefore, it is easy to show that $\chi_r[P_m^{(R)} \odot C_n] = r+1$. □

**Lemma 3.2.** *For any positive integers* $r$, $m, n \geq 2$, *the lower bound for* $r$*-dynamic coloring of* $R$*-vertex corona of path with complete graph* $P_m^{(R)} \odot K_n$ *are*

$$\chi_r[P_m^{(R)} \odot K_n] \geq \begin{cases} n+1, & for\ 1 \leq r \leq n \\ r+1, & for\ n+1 \leq r \leq \Delta \end{cases}$$

**Proof.** Let $V(P_m^{(R)} \odot K_n) = \{u_i, u'_k, v_{ij} : i\epsilon[1, m], j\epsilon[1, n], k\epsilon[1, m-1]\}$, where $u_i$ be the vertices of path graph $P_m$, vertices of $u'_k$ be the vertices of $R$-graph of path $P_m^{(R)}$ corresponding to the vertices of $P_m$ and $v_{ij}$ be the vertices of complete graph $K_n$. In $R$-vertex corona of $P_m^{(R)} \odot K_n$ the m-copies of complete graph $K_n$ are connected to each vertices of $P_m$. The order and the size of the graph $P_m^{(R)} \odot K_n$ are $|V[P_m^{(R)} \odot K_n]| = mn + m + (m-1)$ and $|E[P_m^{(R)} \odot K_n]| = m(\frac{n(n-1)}{2} + n) + (m-1) + 2m - 2$. The minimum and maximum degree are $\delta(P_m^{(R)} \odot K_n) = 2$, $\Delta(P_m^{(R)} \odot K_n) = n+2$ for $m = 2$ and $\Delta(P_m^{(R)} \odot K_n) = n+4$ for $m \geq 2$. For $1 \leq r \leq n$, the vertices $v_{ij}$ persuade a clique of order $n+1$ in $P_m^{(R)} \odot K_n$. Thus, $\chi_r[P_m^{(R)} \odot K_n] \geq n+1$. Then for $n+1 \leq r \leq \Delta$, based on Lemma 2.1, we have $\chi_r[P_m^{(R)} \odot K_n] \geq min\{r, \Delta[P_m^{(R)} \odot K_n]\} + 1 = r+1$. Thus, it completes the proof. □

**Theorem 3.3.** *For any positive integers* $r$, $m, n \geq 2$, *the* $r$*- dynamic coloring of* $R$*-vertex corona of path with complete graph* $P_m^{(R)} \odot K_n$ *are*

$$\chi_r[P_m^{(R)} \odot K_n] = \begin{cases} n+1, & for\ 1 \leq r \leq n \\ r+1, & for\ n+1 \leq r \leq \Delta \end{cases}$$

**Proof.** The $r$-dynamic coloring of $P_m^{(R)} \odot K_n$ are as follows:

**Case :1** $1 \leq r \leq n$

From the lemma 3.2, it is clear that $\chi_r[P_m^{(R)} \odot K_n] \geq n+1$. So it is enough to show that $\chi_r[P_m^{(R)} \odot K_n] \leq n+1$. To determine the value of $\chi_r[P_m^{(R)} \odot K_n] \leq n+1$, consider a function $c_1$, such that $c_1 : V[P_m^{(R)} \odot K_n] \rightarrow \{1, 2, \cdots, k\}$. Then,

$$\begin{aligned} c_1(u_i) &= \{1,2\},\ for\ 1 \leq i \leq m \\ c_1(u'_k) &= 3,\ for\ 1 \leq k \leq m-1 \\ c_1(v_{ij}) &= \{2,3,\cdots,n+1\}\ for\ i\ is\ odd,\ 1 \leq i \leq m,\ 1 \leq j \leq n \\ c_1(v_{ij}) &= \{1,3,4,\cdots,n+1\}\ for\ i\ is\ even,\ 1 \leq i \leq m,\ 1 \leq j \leq n \end{aligned}$$

Thus, $\chi_r[P_m^{(R)} \odot K_n] \leq n+1$. Therefore, it is easy to show that $\chi_r[P_m^{(R)} \odot K_n] = n+1$.

**Case :2** $n+1 \leq r \leq \Delta$

From the lemma 3.2, it is clear that $\chi_r[P_m^{(R)} \odot K_n] \geq r+1$. So it is enough to show that $\chi_r[P_m^{(R)} \odot K_n] \leq r+1$. To determine the value of $\chi_r[P_m^{(R)} \odot K_n] \leq r+1$, consider a function $c_2$, such that $c_2 : V[P_m^{(R)} \odot K_n] \rightarrow \{1, 2, \cdots, k\}$. Then,

- When $n+1 \leq r \leq \Delta - 2$, the $r$-dynamic coloring are as follows:

$$\begin{aligned} c_2(u_i) &= \{1,2\},\ for\ 1 \leq i \leq m \\ c_2(u'_k) &= 3,\ for\ 1 \leq k \leq m-1 \end{aligned}$$

  Color of the vertices $v_{ij}$ from the set of colors $\{4, 5, \cdots, r+1\}$ depending on the $r$-adjacency condition for $1 \leq i \leq m$ and $1 \leq j \leq n$.
  Thus, $\chi_r[P_m^{(R)} \odot K_n] \leq r+1$. From the lemma 3.2, it is easy to show that $\chi_r[P_m^{(R)} \odot K_n] = r+1$.

- When $r = \Delta - 1$, the $r$-dynamic coloring are as follows:

$$\begin{aligned} c_2(u_i) &= \{1,3\},\ for\ 1 \leq i \leq m \\ c_2(u'_k) &= \{2,4\},\ for\ 1 \leq k \leq m-1 \\ c_2(v_{ij}) &= \{5,6,\cdots,r+1\}\ for\ 1 \leq i \leq m,\ 1 \leq j \leq n \end{aligned}$$

  Thus, $\chi_r[P_m^{(R)} \odot K_n] \leq r+1$. From the lemma 3.2, it is proved that $\chi_r[P_m^{(R)} \odot K_n] = r+1$.

- When $r = \Delta$, the $r$-dynamic coloring are as follows:

$$\begin{aligned} c_2(u_i) &= \{1,3,5\},\ for\ 1 \leq i \leq m \\ c_2(u'_k) &= \{2,4\},\ for\ 1 \leq k \leq m-1 \\ c_2(v_{ij}) &= \{6,7,\cdots,r+1\}\ for\ 1 \leq i \leq m,\ 1 \leq j \leq n \end{aligned}$$

  Thus, $\chi_r[P_m^{(R)} \odot K_n] \leq r+1$. From the lemma 3.2, it is easy to show that $\chi_r[P_m^{(R)} \odot K_n] = r+1$. □

**Theorem 3.4.** *For any positive integers $r$, $m \geq 3$, $n \geq 2$, the $r$- dynamic coloring of $R$-vertex corona of cycle with path $C_n^{(R)} \odot P_m$ are*

$$\chi_r[C_n^{(R)} \odot P_m] = \begin{cases} 3, & for\ 1 \leq r \leq 2 \\ r+1, & for\ 3 \leq r \leq \Delta - 1 \\ r+1, & for\ r \geq \Delta,\ n\ is\ even \\ r+2, & for\ r \geq \Delta,\ n\ is\ odd \end{cases}$$

**Proof.** Let us suppose that $V[C_n^{(R)} \odot P_m] = \{u_i, u'_i, v_{ij} : i\epsilon[1,n], j\epsilon[1,m]\}$, where $u_i$ be the vertices of cycle $C_n$, $u'_i$ be the vertices of $R$-graph of cycle $C_n^{(R)}$ and $v_{ij}$ be the vertices of $P_m$

corresponding to the vertices of cycle $C_n$. In $R$-vertex corona of $C_n^{(R)} \odot P_m$ the n-copies of $P_m$ are connected to every vertices of cycle $C_n$. The order and the size of the graph $C_n^{(R)} \odot P_m$ are $|V[C_n^{(R)} \odot P_m]| = mn + 2n$ and $|E[C_n^{(R)} \odot P_m]| = n[m + (m-1)] + 3n$. The minimum and maximum degree are $\delta(C_n^{(R)} \odot P_m) = 2$ and $\Delta(C_n^{(R)} \odot P_m) = m + 4$.

**Case :1** $1 \leq r \leq 2$

To determine the $r$-dynamic coloring of $C_n^{(R)} \odot P_m$, we must show that $\chi_r[C_n^{(R)} \odot P_m] \geq 3$ and $\chi_r[C_n^{(R)} \odot P_m] \leq 3$. To prove the upper bound $\chi_r[C_n^{(R)} \odot P_m] \leq 3$, consider a function $c_1$, such that $c_1 : V[C_n^{(R)} \odot P_m] \to \{1, 2, \cdots, k\}$.

When $n$ is odd,

$$\begin{aligned} c_1(u_i) &= \{1, 2\},\ for\ 1 \leq i \leq n-1 \\ c_1(u_n) &= 3 \\ c_1(u'_i) &= 3,\ for\ 1 \leq i \leq n-2 \\ c_1(u'_{n-1}) &= 1 \\ c_1(u'_n) &= 2 \end{aligned}$$

Color the vertices $v_{ij}$ with the colors from the set $\{1, 2, 3\}$ according to the coloring of the vertices $u_i$. Thus, $\chi_r[C_n^{(R)} \odot P_m] \leq 3$. From the lemma 2.1, it is clear that $\chi_r[C_n^{(R)} \odot P_m] \geq 3$. Hence, it is easy to show that $\chi_r[C_n^{(R)} \odot P_m] = 3$.

When $n$ is even,

$$\begin{aligned} c_1(u_i) &= \{1, 2\},\ for\ 1 \leq i \leq n \\ c_1(u'_i) &= 3,\ for\ 1 \leq i \leq n \\ c_1(v_{ij}) &= \{2, 3\},\ for\ i\ is\ odd,\ 1 \leq i \leq n,\ 1 \leq j \leq m \\ c_1(v_{ij}) &= \{1, 3\},\ for\ i\ is\ even,\ 1 \leq i \leq n,\ 1 \leq j \leq m \end{aligned}$$

Thus, $\chi_r[C_n^{(R)} \odot P_m] \leq 3$. From the lemma 2.1, it is clear that $\chi_r[C_n^{(R)} \odot P_m] \geq 3$. Hence, it is clearly proved that $\chi_r[C_n^{(R)} \odot P_m] = 3$.

**Case :2** $3 \leq r \leq \Delta - 1$

To determine the $r$-dynamic coloring of $C_n^{(R)} \odot P_m$, we must show that $\chi_r[C_n^{(R)} \odot P_m] \geq r + 1$ and $\chi_r[C_n^{(R)} \odot P_m] \leq r + 1$. To prove the upper bound $\chi_r[C_n^{(R)} \odot P_m] \leq r + 1$, consider a function $c_2$, such that $c_2 : V[C_n^{(R)} \odot P_m] \to \{1, 2, \cdots, k\}$.

$$\begin{aligned} c_2(u_i) &= \{1, 2, \cdots, n\},\ for\ 1 \leq i \leq n \\ c_2(v_{ij}) &= \{1, 2, \cdots, r\},\ for\ 1 \leq i \leq n,\ 1 \leq j \leq m \end{aligned}$$

The coloring of the vertices $v_{ij}$ depends on the coloring of the vertices $u_i$, since the $n$-copies of $v_{ij}$ are connected to each vertices of $u_i$. Finally, we have

$$c_2(u'_i) = r + 1,\ for\ 1 \leq i \leq n$$

Thus, $\chi_r[C_n^{(R)} \odot P_m] \leq r + 1$. From the lemma 2.1, it is clear that $\chi_r[C_n^{(R)} \odot P_m] \geq r + 1$. Hence, it is clearly proved that $\chi_r[C_n^{(R)} \odot P_m] = r + 1$.

**Case :3** $r \geq \Delta$

Consider a function $c_3$, such that $c_3 : V[C_n^{(R)} \odot P_m] \to \{1, 2, \cdots, k\}$.

- When $n$ is even, to determine the $r$-dynamic coloring of $C_n^{(R)} \odot P_m$, we must show that $\chi_r[C_n^{(R)} \odot P_m] \geq r + 1$ and $\chi_r[C_n^{(R)} \odot P_m] \leq r + 1$. To prove the upper bound $\chi_r[C_n^{(R)} \odot P_m] \leq r + 1$, consider the following coloring:

$$\begin{aligned} c_3(u_i) &= \{1, 2, \cdots, n\},\ for\ 1 \leq i \leq n \\ c_3(v_{ij}) &= \{1, 2, \cdots, r-1\},\ for\ 1 \leq i \leq n,\ 1 \leq j \leq m \\ c_3(u'_i) &= \{r, r+1\}\ for\ 1 \leq i \leq n \end{aligned}$$

Thus, $\chi_r[C_n^{(R)} \odot P_m] \leq r+1$. From the lemma 2.1, it is clear that $\chi_r[C_n^{(R)} \odot P_m] \geq r+1$. Hence, it is clearly proved that $\chi_r[C_n^{(R)} \odot P_m] = r+1$.

- When $n$ is odd, to determine the $r$-dynamic coloring of $C_n^{(R)} \odot P_m$, we must show that $\chi_r[C_n^{(R)} \odot P_m] \geq r+2$ and $\chi_r[C_n^{(R)} \odot P_m] \leq r+2$. To prove the upper bound $\chi_r[C_n^{(R)} \odot P_m] \leq r+2$, consider the following coloring:

$$\begin{aligned} c_3(u_i) &= \{1, 2, \cdots, n\},\ for\ 1 \leq i \leq n \\ c_3(v_{ij}) &= \{1, 2, \cdots, r-1\},\ for\ 1 \leq i \leq n,\ 1 \leq j \leq m \end{aligned}$$

The coloring of the vertices $v_{ij}$ depends on the coloring of the vertices $u_i$, since the $n$-copies of $v_{ij}$ are connected to each vertices of $u_i$. Atlast, we have

$$c_3(u_i') = \{r, r+1, r+2\}\ for\ 1 \leq i \leq n$$

Thus, $\chi_r[C_n^{(R)} \odot P_m] \leq r+2$. From the lemma 2.1, it is clear that $\chi_r[C_n^{(R)} \odot P_m] \geq r+2$. Hence, it is clearly proved that $\chi_r[C_n^{(R)} \odot P_m] = r+2$.□

**Lemma 3.5.** *For any positive integers $r$, $m, n \geq 2$, the lower bound for $r$-dynamic coloring of $R$-vertex corona of complete graph with path $K_n^{(R)} \odot P_m$ are*

$$\chi_r[K_n^{(R)} \odot P_m] \geq \begin{cases} n, & for\ 1 \leq r \leq n-1 \\ r+1, & for\ n \leq r \leq \Delta \end{cases}$$

**Proof.** Let $V(K_n^{(R)} \odot P_m) = \{v_i, v_i', u_{ij} : i\epsilon[1, n], j\epsilon[1, m]\}$, where $v_i$ be the vertices of path graph $K_n$, vertices of $v_i'$ be the vertices of $R$-graph of path $K_n^{(R)}$ and $u_{ij}$ be the vertices of path $P_m$ connected to every vertices of complete graph $K_n$. The order of the graph are $|V[K_n^{(R)} \odot P_m]| = (mn+2n)-1$ and the size of the graph are $|E[K_n^{(R)} \odot P_m]| = \frac{n(n-1)}{2} + 2n + n(2m-1) - 2$ for $n = 2$. When $n \geq 3$, the order of the graph are $|V[K_n^{(R)} \odot P_m]| = mn + 2n$ and the size of the graph are $|E[K_n^{(R)} \odot P_m]| = \frac{n(n-1)}{2} + 2n + n(2m-1)$. The minimum and maximum degree are $\delta(K_n^{(R)} \odot P_m) = 2$, $\Delta(K_n^{(R)} \odot P_m) = m+n$ for $n = 2$ and $\Delta(K_n^{(R)} \odot P_m) = m+n+1$ for $n \geq 3$.

For $1 \leq r \leq n-1$, the vertices $v_i$ persuade a clique of order $n$ in $K_n^{(R)} \odot P_m$. Thus, $\chi_r[K_n^{(R)} \odot P_m] \geq n$. Then for $n \leq r \leq \Delta$ based on Lemma 2.1, we have $\chi_r[K_n^{(R)} \odot P_m] \geq min\{r, \Delta[K_n^{(R)} \odot P_m]\} + 1 = r+1$. Thus, it completes the proof.□

**Theorem 3.6.** *For any positive integers $r$, $m, n \geq 2$, the $r$- dynamic coloring of $R$-vertex corona of complete graph with path $K_n^{(R)} \odot P_m$ are*

$$\chi_r[K_n^{(R)} \odot P_m] = \begin{cases} n, & for\ 1 \leq r \leq n-1 \\ r+1, & for\ n \leq r \leq \Delta - 1 \\ r+1, & for\ r \geq \Delta\ n\ is\ even \\ r+2, & for\ r \geq \Delta\ n\ is\ odd \end{cases}$$

**Proof.** The $r$-dynamic coloring of $K_n^{(R)} \odot P_m$ are consider in the following cases:

**Case :1** $1 \leq r \leq n-1$

From the lemma 3.5, it is clear that $\chi_r[K_n^{(R)} \odot P_m] \geq n$. So it is enough to show that $\chi_r[K_n^{(R)} \odot P_m] \leq n$. To indicate the upper bound $\chi_r[K_n^{(R)} \odot P_m] \leq n$, consider a function $c_1$, such that $c_1 : V[K_n^{(R)} \odot P_m] \to \{1, 2, \cdots, k\}$. Then,

$$\begin{aligned} c_1(v_i) &= \{1, 2, \cdots, n\},\ for\ 1 \leq i \leq n \\ c_1(v_i') &= \{3, 4, \cdots, n, 1, 2\}\ for\ 1 \leq i \leq n \end{aligned}$$

Similarly, color the vertices $c_1(u_{ij}) = \{1, 2, \cdots, n\}$ such that the color of the vertex $v_i$ should not adjacent to the color of the vertices $u_{ij}$ for $1 \leq i \leq n$ and $1 \leq j \leq m$. Thus, $\chi_r[K_n^{(R)} \odot P_m] \leq n$. Therefore, it is easy to prove that $\chi_r[K_n^{(R)} \odot P_m] = n$.

**Case :2** $n \leq r \leq \Delta - 1$

From the lemma 3.5, it is easy to show that $\chi_r[K_n^{(R)} \odot P_m] \geq r + 1$. So it is enough to prove that $\chi_r[K_n^{(R)} \odot P_m] \leq r + 1$. To indicate the upper bound $\chi_r[K_n^{(R)} \odot P_m] \leq r + 1$, consider a function $c_2$, such that $c_2 : V[K_n^{(R)} \odot P_m] \to \{1, 2, \cdots, k\}$.

- When $n \leq r \leq \Delta - 2$,

$$\begin{aligned} c_2(v_i) &= \{1, 2, \cdots, n\},\ for\ 1 \leq i \leq n \\ c_2(v_i') &= \{3, 4, \cdots, n, 1, 2\}\ for\ 1 \leq i \leq n \end{aligned}$$

  If $n \leq r \leq \Delta - 3$, the coloring of the vertices $u_{ij}$ may varies according to the $r$-adjacency condition. So, the vertices $u_{ij}$ are colored from the set $\{1, 2, \cdots, r + 1\}$, else the vertices $u_{ij}$ may color from the set $\{n+1, n+2, \cdots, r+1\}$, when $r = \Delta - 2$.

- When $r = \Delta - 1$,

$$\begin{aligned} c_2(v_i) &= \{1, 2, \cdots, n\},\ for\ 1 \leq i \leq n \\ c_2(u_{ij}) &= \{n + 1, n + 2, \cdots, r\}\ for\ 1 \leq i \leq n\ 1 \leq j \leq m \\ c_2(v_i') &= r + 1\ for\ 1 \leq i \leq n \end{aligned}$$

  Thus, $\chi_r[K_n^{(R)} \odot P_m] \leq r+1$. Therefore, it is easy to prove that $\chi_r[K_n^{(R)} \odot P_m] = r+1$.

**Case :3** $r \geq \Delta$

Consider a function $c_3$, such that $c_3 : V[K_n^{(R)} \odot P_m] \to \{1, 2, \cdots, k\}$.

- When $n$ is even, from the lemma 3.5, it is easy to show that $\chi_r[K_n^{(R)} \odot P_m] \geq r + 1$. So to determine the upper bound $\chi_r[K_n^{(R)} \odot P_m] \leq r + 1$, consider the below coloring:

$$\begin{aligned} c_3(v_i) &= \{1, 2, \cdots, n\},\ for\ 1 \leq i \leq n \\ c_3(u_{ij}) &= \{n + 1, n + 2, \cdots, r - 1\}\ for\ 1 \leq i \leq n\ 1 \leq j \leq m \\ c_3(v_i') &= \{r, r + 1\}\ for\ 1 \leq i \leq n \end{aligned}$$

  Thus, $\chi_r[K_n^{(R)} \odot P_m] \leq r + 1$. Hence, it is proved that $\chi_r[K_n^{(R)} \odot P_m] = r + 1$.

- When $n$ is odd, from the lemma 3.5, it is easy to show that $\chi_r[K_n^{(R)} \odot P_m] \geq r+2$. So to determine the upper bound $\chi_r[K_n^{(R)} \odot P_m] \leq r + 2$, consider the below coloring:

$$\begin{aligned} c_3(v_i) &= \{1, 2, \cdots, n\},\ for\ 1 \leq i \leq n \\ c_3(u_{ij}) &= \{n + 1, n + 2, \cdots, r\}\ for\ 1 \leq i \leq n\ 1 \leq j \leq m \\ c_3(v_i') &= \{r, r + 1, r + 2\}\ for\ 1 \leq i \leq n \end{aligned}$$

  Thus, $\chi_r[K_n^{(R)} \odot P_m] \leq r + 2$. Therefore, it is proved that $\chi_r[K_n^{(R)} \odot P_m] = r + 2$.□

## Conclusion

In this paper, we have extract the exact results of $r$-dynamic coloring of $R$-vertex corona of some graphs such as path with cycle, path with complete graph, cycle with path and complete graph with path.

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