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ON DECOMPOSITIONS OF θg^*s-CLOSED SETS IN TOPOLOGICAL SPACES

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Abstract The primary aim of this paper is to introduce and study the classes of θg^*s-locally closed set and different notions of generalization of continuous functions namely θg^*slc-set, θg^*slc^*-set, θg^*slc^{**}-set, θg^*slc-continuity, θg^*slc^*-continuity and θg^*slc^{**}-continuity and investigates some of their properties. Also, we have given an appropriate examples to understand the abstract concepts clearly. Furthermore, the notions of GB-sets, GB_s-sets and GB_{s^*}-sets were acquires. Employing these notions, decompositions of θg^*s-closed sets are also obtain.

1 Introduction

Continuity of functions is one of the core principles of topology. Many different kinds of continuous functions have been launched through the years. Various interesting problems arise when one considers continuity or a stronger form of continuity or a weaker form of continuity. One of them, that can be of great interest to general topologists, is its decompositions. In 1986, Tong [14] obtained a decomposition of continuity in topological spaces. In 1986 & in 1989 Jingcheng Tong [13, 14] introduced two classes of sets, namely A-set & B-set and using them obtained decomposition of continuity.

The first step of locally closedness was done by [3]. He defined a set A to be locally closed if it is the intersection of an open and a closed set. Extensive research on locally closedness and generalizing locally closedness were done in recent years. Stone [11] used the term LC for a locally closed set. In this paper, we define the notions of θg^*slc-continuity, θg^*slc^*-continuity and θg^*slc^{**}-continuity and investigates some of their properties. Also, we have given an appropriate examples to understand the abstract concepts clearly. Furthermore, the notions of GB-sets, GB_s-sets and GB_{s^*}-sets were acquires. Employing these notions, decompositions of θg^*s-closed sets are also obtain.

2 Preliminaries

The following recalls requisite ideas and preliminaries necessitated in the sequel of our work.

Definition 2.1. [5] A subset S of a space (X, τ) is called locally closed(briefly lc) if $S = U \cap F$, Where U is open and F is closed in (X, τ).

Definition 2.2. [7] A subset A of a space X is said to be dense in X if $cl(A) = X$.

Definition 2.3. [8] A subset A of a space X is said to be θ-dense in X if $cl_\theta(A) = X$.

Definition 2.4. [1] A set A of (X, τ) is called θ-locally closed If $A = S \cap F$, Where S is θ-open and F is θ-closed.

Definition 2.5. [1] A set A is called
(1) θlc^*-set If $A = S \cap F$, Where S is θ-open and F is closed.
(2) θlc^{**}-set If $A = S \cap F$, Where S is open and F is θ-closed.

Definition 2.6. [1] A subset S of a topological space (X, τ) is called a
(1) generalized locally closed (briefly glc) [2] $if S = U \cap F$, where U is g-open and F is g-closed in (X, τ).
(2) θ-generalized locally closed (θglc)-set [1] If $S = P \cap F$ Where P is θg-open and F is θg-closed.
(3) θglc^*-set[1] If $S = P \cap F$ Where P is θg-open and F is θ-closed.
(4) θglc^{**}-set[1] If $S = P \cap F$ Where P is θ-open and F is θg-closed.

Remark 2.7. [12] (1)Every r-closed in X is $\theta g^* s$-closed.
(2)Every closed set is $\theta g^* s$-closed.
(3)Every $\theta g^* s$-closed is rg-closed.

Definition 2.8. A subset S of a space (X, τ) is called
(1) submaximal [4] if every dense subset is open.
(2) g-submaximal [3] if every dense subset is g-open.
(3) sg-submaximal [6] if every dense subset of X is sg-open.

Definition 2.9. A subset S of a space (X, τ) is called
(1) an α^*-set [9] if $int(A) = int(cl(int(A)))$.
(2) an A-set [13] if $A = G \cap F$ Where G is open and F is regular closed in X.
(3) a t-set [14] if $int(A) = int(cl(A))$.
(4) a C-set [10] if $A = G \cap F$ Where G is open and F is a t-set in X.
(5) a C_r-set [10] if $A = G \cap F$ Where G is rg-open and F is a t-set in X.
(6) a C_{r^*}-set [10] if $A = G \cap F$ Where G is rg-open and F is an α^*-set in X.

Definition 2.10. [12] A subset A of a topological space (X, τ) is called θ-generalized star semi-closed (briefly $\theta g^* s$-closed) if $scl_\theta(A) \subseteq U$ whenever $A \subseteq U$ and U is g-open. The complement of $\theta g^* s$-closed set is called $\theta g^* s$-open.

Definition 2.11. [12] For a subset A of a space X, $\theta g^* s\text{-}cl(A) = \bigcap\{F : A \subseteq F, F$ is $\theta g^* s$-closed in $X\}$ is called the $\theta g^* s$-closure of A.

3 $\theta g^* s$-locally closed sets

In this section, we define and study the notions of $\theta g^* slc$-set, $\theta g^* slc^*$-set and $\theta g^* slc^{**}$-set in topological spaces.

Definition 3.1. A subset A of (X, τ) is said to be θ-generalized star semi locally closed set(briefly, $\theta g^* slc$) if $S = L \cap M$, Where L is $\theta g^* s$-open and M is $\theta g^* s$-closed.

Definition 3.2. A subset A of (X, τ) is said to be $\theta g^* slc^*$-set if there exists a $\theta g^* s$-open set L and a closed set M of (X, τ) such that $S = L \cap M$.

Definition 3.3. A subset A of (X, τ) is said to be $\theta g^* slc^{**}$-set if there exists an open set L and a $\theta g^* s$ closed set M such that $B = L \cap M$.

Proposition 3.4. *(1) Every locally closed set is $\theta g^* slc$.*
(2) Every θlc is $\theta g^* slc$.
(3) Every $\theta g^* slc^*$-set is $\theta g^* slc$.
(4) Every $\theta g^* slc^{**}$-set is $\theta g^* slc$.
(5) Every $\theta g^* slc$-set is glc.
(6) Every θlc-set is $\theta g^* slc^*$.
(7) Every θlc-set is $\theta g^* slc^{**}$.
(8) Every θlc^*-set is $\theta g^* slc$.
(9) Every θlc^*-set is $\theta g^* slc^{**}$.
(10) Every θlc^{**}-set is $\theta g^* slc$.
(11) Every θglc^*-set is $\theta g^* slc$.
(12) Every $\theta g^* slc$-set is θglc^{**}.
(13) Every $\theta g^* s$-closed set is $\theta g^* slc$.

However the converses of the above are not true as seen by the following examples

Example 3.5. Let X = {a, b, c} with τ = {ϕ, {a, b}, X}. Then subset A = {b} is $\theta g^* slc$-set but not locally closed.

Example 3.6. In Example 3.5, Let subset $A = \{c\}$ is $\theta g^* slc$-set but not θlc-set.

Example 3.7. In Example 3.5, Let subset $A = \{b, c\}$ is $\theta g^* slc$-set but not $\theta g^* slc^*$-set.

Example 3.8. In Example 3.5, Let subset $A = \{b\}$ is $\theta g^* slc$-set but not $\theta g^* slc^{**}$-set.

Example 3.9. Let X = {a, b, c} with τ = {ϕ, {c}, {a, b}, X}. Then subset A = {a} is glc-set but not $\theta g^* slc$-set.

Example 3.10. In Example 3.5, Let subset $A = \{a\}$ is $\theta g^* slc^*$-set but not θlc-set.

Example 3.11. In Example 3.5, Let subset $A = \{a, b\}$ is $\theta g^* slc^{**}$-set but not θlc-set.

Example 3.12. In Example 3.5, Let subset $A = \{b, c\}$ is $\theta g^* slc$-set but not θlc^*-set.

Example 3.13. Let X = {a, b, c, d} with τ = {ϕ, {d}, X}. Then subset A = {d} is $\theta g^* slc^{**}$-set but not θlc^*-set.

Example 3.14. In Example 3.5, Let subset $A = \{b\}$ is $\theta g^* slc$-set but not θlc^{**}-set.

Example 3.15. In Example 3.5, Let subset $A = \{b, c\}$ is $\theta g^* slc$-set but not θglc^*-set.

Example 3.16. In Example 3.13, Let subset $A = \{a, d\}$ is θglc^{**}-set but not $\theta g^* slc$-set.

Example 3.17. Let X = {a, b, c} with τ = {ϕ, {c}, X}. Let subset A = {c} is $\theta g^* slc$-set but not $\theta g^* s$-closed set.

Remark 3.18. The concepts of $\theta g^* slc^*$ and $\theta g^* slc^{**}$ are independent of each other.

Example 3.19. In Example 3.5, Let subset $A = \{a\}$ is $\theta g^* slc^*$-set but not $\theta g^* slc^{**}$-set and Let subset $A = \{a, c\}$ is $\theta g^* slc^{**}$-set but not $\theta g^* slc^*$-set.

Remark 3.20. The concepts of $\theta g^* slc^*$ and θlc^{**} are independent of each other.

Example 3.21. Let X = {a, b, c} with τ = {ϕ, {a}, {a, b}, {a, c}, X}. Let subset A = {b} is $\theta g^* slc^*$-set but not θlc^{**}-set and Let subset A = {a, b} is θlc^{**}-set but not $\theta g^* slc^*$-set.

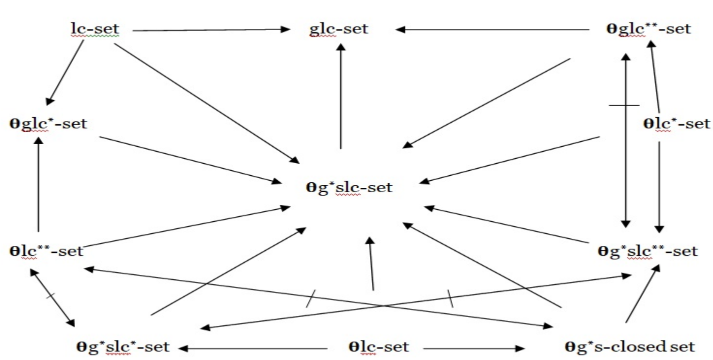
Remark 3.22. The concepts of $\theta g^* slc^{**}$ and θglc^{**} are independent of each other.

Example 3.23. In Example 3.13, Let subset $A = \{d\}$ is $\theta g^* slc^{**}$-set but not θglc^{**}-set and Let subset $A = \{a, d\}$ is θglc^{**}-set but not $\theta g^* slc^{**}$-set.

Remark 3.24. The concepts of θlc^{**}-set and $\theta g^* s$-closed set are independent of each other.

Example 3.25. In Example 3.17, Let subset $A = \{c\}$ is θlc^{**}-set but not $\theta g^* s$-closed set and Let subset $A = \{a, b\}$ is $\theta g^* s$-closed set but not θlc^{**}-set .

Remark 3.26. The above discussions are summarized in the following implications.



None of the implications are reversible.

Remark 3.27. Union of any two θg^*slc-sets are θg^*slc-sets.

Remark 3.28. Union of any two θg^*slc^*-sets need not be an θg^*slc^*-set as seen from the following example.

Example 3.29. Let X = {a, b, c} with τ = {ϕ, {b}, {a, b}, {b, c}, X}. Then the subsets {a} and {b} are θg^*slc^*-sets but their union {a, b} $\notin \theta g^*slc^*(X)$.

Remark 3.30. Intersection of any two θg^*slc^{**}-set need not be an θg^*slc^{**}-set as seen from the following example.

Example 3.31. In Example 3.5, Let the subsets $\{a, c\}$ and $\{a, b\}$ are θg^*slc^{**}-set but their intersection $\{a\} \notin \theta g^*slc^{**}(X)$.

4 θg^*s-DENSE SETS AND θg^*s-SUBMAXIMAL SPACES

In this section, we introduce and study the concept of θg^*s-dense set and θg^*s-submaximal spaces are defined and their properties are obtained.

Definition 4.1. A subset A of (X, τ) is called θg^*s-dense if $\theta g^*s\text{-}cl(A) = X$.

Remark 4.2. A subset S of a topological space (X, τ) is called a
(1) θg^*s-closed iff $\theta g^*s\text{-}cl(A) = A$.
(2) θg^*s-open iff $\theta g^*s\text{-}int(A) = A$.

Example 4.3. Let X = {a, b, c} with τ = {ϕ, {a, b}, X}. Then subset A = {a, b} is θg^*s-dense in (X, τ).

Proposition 4.4. *Every θg^*s-dense set is θ-dense and dense.*

Proof. Let A be θg^*s-dense set in X. Then $\theta g^*s\text{-}cl(A) = X$. Since $\theta g^*s\text{-}cl(A) \subseteq \theta cl(A) \subseteq cl(A)$. we have $cl(A) = X$ and so A is θ-dense and dense. The converse of the above proposition need not be true as seen from the following example.

Example 4.5. In Example 4.3, Let subset $A = \{b, c\}$ is a dense in X but it is not θg^*s-dense in (X, τ).

Definition 4.6. A topological space (X, τ) is called θg^*s-submaximal if every dense subset in it is θg^*s-open in (X, τ)

Proposition 4.7. *Every submaximal space is θg^*s-submaximal.*

Proof. Let (X, τ) be a submaximal space and A be a dense subset of (X, τ). Then A is open. But every open set is θg^*s-open and so A is θg^*s-open. Therefore (X, τ) is θg^*s-submaximal.
The converse of the above proposition need not be true as seen from the following example.

Example 4.8. Let X = {a, b, c} with τ = {ϕ, {c}, X}. Then $\theta g^*sO(X) = P(X)$. we have every dense subset is θg^*s-open and hence (X, τ) is θg^*s-submaximal. However, the set A = {b, c} is dense in (X, τ), but it is not open in (X, τ). Therefore (X, τ) is not submaximal.

Proposition 4.9. *Every θg^*s-submaximal space is g-submaximal.*

Proof. Let (X, τ) be a θg^*s-submaximal space and A be a dense subset of (X, τ). Then A is θg^*s-open. But every θg^*s-open set is g-open and A is g-open. Therefore (X, τ) is g-submaximal.
The converse of the above proposition need not be true as seen from the following example.

Example 4.10. In Example 4.8, Let $GO(X) = P(X)$ and $\theta g^*sO(X)$ = {ϕ, {c}, X} we have every dense subset is g-open and hence (X, τ) is g-submaximal. However, the set A = {b, c} is dense in (X, τ), but it is not θg^*s-open in (X, τ). Therefore (X, τ) is not θg^*s-submaximal.

Proposition 4.11. *Every θg^*s-submaximal space is sg-submaximal.*

Proof. Let (X, τ) be a θg^*s-submaximal space and A be a dense subset of (X, τ). Then A is θg^*s-open. But every θg^*s-open set is sg-open and A is sg-open. Therefore (X, τ) is sg-submaximal The converse of the above proposition need not be true as seen from the following example.

Example 4.12. Let X = {a, b, c} with τ = {ϕ, {c}, {a, b}, X}. Then $SGO(X) = P(X)$ and $\theta g^*sO(X)$ = {ϕ, {c}, {a, b}, X}. Every dense subset is sg-open and hence (X, τ) is sg-submaximal. However, the set A = {a, c} is dense in (X, τ), but it is not θg^*s-open in (X, τ). Therefore (X, τ) is not θg^*s-submaximal.

Remark 4.13. From propositions 4.7, 4.9 and 4.11, we have the following diagram.
Submaximal $\longrightarrow$ θg^*s-submaximal $\longrightarrow$ g-submaximal $\longrightarrow$ sg-submaximal

Theorem 4.14. *For a subset A of (X, τ) the following statements are equivalent.*
(1) $A \in \theta g^*slc(X)$.
(2) $A = S \cap \theta g^*s\text{-}cl(A)$ for some θg^*s-open set S.
(3) $\theta g^*s\text{-}cl(A) - A$ is θg^*s-closed.
(4) $A \cup (\theta g^*s\text{-}cl(A))^c$ is θg^*s-open.
(5) $A \subseteq \theta g^*s\text{-}int(A \cup (\theta g^*s\text{-}cl(A))^c)$.

Proof.(1 $\Rightarrow$ 2) Let $A \in \theta g^*slc(X)$. Then $A = S \cap G$ where S is θg^*s-open and G is θg^*s-closed. Since $A \subseteq G$, $\theta g^*s\text{-}cl(A) \subseteq G$ and so $S \cap \theta g^*s\text{-}cl(A) \subseteq A$. Also $A \subseteq S$ and $A \subseteq \theta g^*s\text{-}cl(A)$ implies $A \subseteq S \cap \theta g^*s\text{-}cl(A)$ and therefore $A = S \cap \theta g^*s\text{-}cl(A)$.

(2 $\Rightarrow$ 3) $A = S \cap \theta g^*s\text{-}cl(A)$ implies $\theta g^*s\text{-}cl(A) - A = \theta g^*s\text{-}cl(A) \cap S^c$ which is θg^*s-closed since S^c is θg^*s-closed and $\theta g^*s\text{-}cl(A)$ is θg^*s-closed.

(3 $\Rightarrow$ 4) $A \cup (\theta g^*s\text{-}cl(A))^c = (\theta g^*s\text{-}cl(A) - A)^c$ and by assumption, $(\theta g^*s\text{-}cl(A) - A)^c$ is θg^*s-open and so is $A \cup (\theta g^*s\text{-}cl(A))^c$ is θg^*s-open. (4 $\Rightarrow$ 5) By assumption, $A \cup (\theta g^*s\text{-}cl(A))^c = \theta g^*s\text{-}int(A \cup (\theta g^*s\text{-}cl(A))^c)$ and hence $A \subseteq \theta g^*s\text{-}int(A \cup (\theta g^*s\text{-}cl(A))^c)$.

(5 $\Rightarrow$ 1) By assumption and since $A \subseteq \theta g^*s\text{-}cl(A)$, $A = \theta g^*s\text{-}int(A \cup (\theta g^*s\text{-}cl(A))^c) \cap \theta g^*s\text{-}cl(A)$. Therefore, $A \in \theta g^*lc(X)$.

Theorem 4.15. *For a subset A of (X, τ), the following statements are equivalent*
(1) $A \in \theta g^*slc^*(X)$.
(2) $A = S \cap cl(A)$ for some θg^*s-open set S.
(3) $cl(A) - A$ is θg^*s-closed.
(4) $A \cup (cl(A))^c$ is θg^*s-open.

Proof. (1 $\Rightarrow$ 2) Let $A \in \theta g^*slc^*(X)$. There exist an θg^*s-open set S and a closed set R such that $A = S \cap R$. Since $A \subseteq S$ and $A \subseteq cl(A)$, $A \subseteq S \cap cl(A)$. Also since $cl(A) \subseteq R$, $S \cap cl(A) \subseteq S \cap R = A$. Therefore $A = S \cap cl(A)$.

(2 $\Rightarrow$ 1) Since A is θg^*s-open and $cl(A)$ is a closed set, $A = S \cap cl(A) \in \theta g^*slc^*(X)$.

(2 $\Rightarrow$ 3) Since $cl(A) - A = cl(A) \cap S^c$, $cl(A) - A$ is θg^*s-closed by remark 4.2.

(3 $\Rightarrow$ 2) Let $S = (cl(A) - A)^c$. Then by assumption S is θg^*s-open in (X, τ) and $A = S \cap cl(A)$.

(3 $\Rightarrow$ 4) Let $R = cl(A) - A$. Then $R^c = A \cup (cl(A))^c$ and $A \cup (cl(A))^c$ is θg^*s-open.

(4 $\Rightarrow$ 3) Let $S = A \cup (cl(A))^c$. Then S^c is θg^*s-closed and $S^c = cl(A) - A$ and so $cl(A) - A$ is θg^*s-closed.

Theorem 4.16. *A space (X, τ) is θg^*s-submaximal iff $P(X) = \theta g^*slc^*(X)$.*

Proof. Necessity: Let $A \in P(X)$ and let $V = A \cup (cl(A))^c$. This implies that $cl(V) = cl(A) \cup (cl(A))^c = X$. Hence $cl(V) = X$. Therefore V is a dense subset of X. Since (X, τ) is θg^*s-submaximal, V is θg^*s-open. Thus $A \cup (cl(A))^c$ is θg^*s-open and by Theorem 4.15, we have $A \in \theta g^*slc^*(X)$.

Sufficiency: Let A be a dense subset of (X, τ). This implies $A \cup (cl(A))^c = A \cup X^c = A \cup \phi = A$. Now $A \in \theta g^*slc^*(X)$ implies that $A = A \cup (cl(A))^c$ is θg^*s-open by Theorem 4.15. Hence (X, τ) is θg^*s-submaximal.

Theorem 4.17. *Let A be a subset of (X, τ). Then $A \in \theta g^*slc^{**}(X)$ if and only if $A = S \cap \theta g^*s$-$cl(A)$ for some open set S.*

Proof. Let $A \in \theta g^*slc^{**}(X)$ Then $A = S \cap G$ where S is open and G is θg^*s-closed. Since $A \subseteq G, \theta g^*s$-$cl(A) \subseteq G$. We obtain $A = A \cap \theta g^*s$-$cl(A) = S \cap G \cap \theta g^*s$-$cl(A) = S \cap \theta g^*s$-$cl(A)$.
Converse part is trivial.

Theorem 4.18. *Let A be a subset of (X, τ). If $A \in \theta g^*slc^{**}(X)$, then θg^*s-$cl(A) - A$ is θg^*s-closed and $A \cup (\theta g^*s$-$cl(A))^c$ is θg^*s-open.*

Proof. Let $A \in \theta g^*slc^{**}(X)$. Then by Theorem 4.17, $A = S \cap \theta g^*s$-$cl(A)$ for some open set S and θg^*s-$cl(A) - A$ = θg^*s-$cl(A) \cap S^c$ is θg^*s-closed in (X, τ). If $G = \theta g^*s$-$cl(A) - A$, then $G^c = A \cup (\theta g^*s$-$cl(A))^c$ and G^c is θg^*s-open and so is $A \cup (\theta g^*s$-$cl(A))^c$.

Proposition 4.19. *For subsets A and B in (X, τ), the following are true.*
(1) *If $A, B \in \theta g^*slc(X)$, then $A \cap B \in \theta g^*slc(X)$.*
(2) *If $A, B \in \theta g^*slc^*(X)$, then $A \cap B \in \theta g^*slc^*(X)$.*
(3) *If $A, B \in \theta g^*slc^{**}(X)$, then $A \cap B \in \theta g^*slc^{**}(X)$.*
(4) *If $A \in \theta g^*slc(X)$ and B is θg^*s-open (resp. θg^*s-closed), then $A \cap B \in \theta g^*slc(X)$.*
(5) *If $A \in \theta g^*slc^*(X)$ and B is θg^*s-open (resp. closed), then $A \cap B \in \theta g^*slc^*(X)$.*
(6) *If $A \in \theta g^*slc^{**}(X)$ and B is θg^*s-closed (resp. open) then $A \cap B \in \theta g^*slc^{*ast}(X)$.*
(7) *If $A \in \theta g^*slc^*(X)$ and B is θg^*s-closed, then $A \cap B \in \theta g^*slc(X)$.*
(8) *If $A \in \theta g^*slc^{**}(X)$, and B is θg^*s-open, then $A \cap B \in \theta g^*slc(X)$.*
(9) *If $A \in \theta g^*slc^{**}(X)$ and $B \in \theta g^*slc^*(X)$, then $A \cap B \in \theta g^*slc(X)$.*

Proof. By remark 2.7 and remark 4.2, (1) to (8) hold.
(9). Let $A = S \cap G$ where S is open and G is θg^*s-closed and $B = P \cap Q$ where P is θg^*s-open and Q is closed. Then $A \cap B = (S \cap P) \cap (G \cap Q)$ where $S \cap P$ is θg^*s-open and $G \cap Q$ is θg^*s-closed, by remark 4.2. Therefore $A \cap B \in \theta g^*slc(X)$.

5 DECOMPOSITION OF θg^*s-CLOSED SET

In this section, we introduce the notions of GB-sets, GB_S-sets and GB_{s^*}-sets to obtain the decompositions of θg^*s-closed sets.

Definition 5.1. A subset S of (X, τ) is called a
(1) GB-set if $M = P \cap Q$, where P is θg^*s-open and Q is a t-set.
(2) GB_s-set if $M = P \cap Q$, where P is θg^*s-open and Q is a α^*-set.
(3) GB_{s^*}-set if $M = P \cap Q$, where P is θg^*s-open and Q is a A-set.

Proposition 5.2. *Let (X, τ) be the topological spaces. Then*
(1) *Every GB-set is a C-set.*
(2) *Every GB-set is a C_r-set.*
(3) *Every GB-set is a C_{r^*}-set.*
(4) *Every GB_s-set is a C-set.*
(5) *Every GB_s-set is a C_r-set.*
(6) *Every GB_s-set is a C_{r^*}-set.*
(7) *Every GB_{s^*}-set is a C-set.*
(8) *Every GB_{s^*}-set is a C_r-set.*
(9) *Every GB_{s^*}-set is a C_{r^*}-set.*
(10) *Every GB_{s^*}-set is a GB-set.*
(11) *Every GB_{s^*}-set is a GB_s-set.*
(12) *Every A-set is a GB-set.*
(13) *Every A-set is a GB_s-set.*
(14) *Every GB-set is a GB_s-set.*
(15) *Every t-set is a GB_s-set.*

Remark 5.3. The converse of the above proposition need not be true as seen from the following examples.

Example 5.4. Let X = {a, b, c} with τ = {ϕ, {b}, {a, b}, {b, c}, X}. Then subset A = {a, b} is a C-set but not GB-set and GB_s-set.

Example 5.5. Let X = {a, b, c} with τ = {ϕ, {c}, X}. Let subset A = {a, c}. Then A is a C_r-set, C_{r^*}-set but A is not a GB-set.

Example 5.6. In Example 5.5, Let subset $A = \{a, c\}$. Then A is a C_r-set, C_{r^*}-set but A is not a GB_s-set.

Example 5.7. In Example 5.5, Let subset $A = \{a, c\}$. Then A is a C-set, C_r-set, C_{r^*}-set but A is not a GB_{s^*}-set.

Example 5.8. In Example 5.5, Let subset $A = \{a, b\}$. Then A is a GB-set, GB_s-set but A is not a GB_{s^*}-set and A-set.

Example 5.9. Let X = {a, b, c} with τ = {ϕ, {a, b}, X}. Then subset A = {a, c} is GB_s-set but A is not a GB-set and A-set.

Example 5.10. In Example 5.9, Let subset $A = \{a, c\}$. Then A is a GB_s-set but A is not a t-set.

Remark 5.11. The concepts of GB-set and α^*-set are independent of each other as seen from the following example.

Example 5.12. In Example 5.9, Let subset $A = \{a, b\}$ is GB-set but not α^*-set and let $A = \{b, c\}$ is a α^*-set but not GB-set.

Remark 5.13. The concepts of GB_{s^*}-set and t-set are independent of each other as seen from the following example.

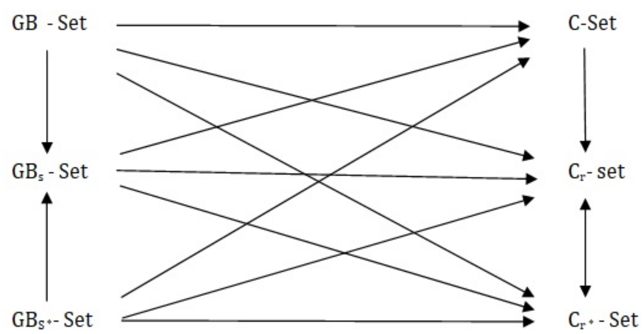
Example 5.14. In Example 5.5, Let subset $A = \{c\}$ is GB_{s^*}-set but not t-set and let $A = \{a, b\}$ is a t-set but not GB_{s^*}-set.

Remark 5.15. The concepts of GB_{s^*}-set and α^*-set are independent of each other as seen from the following example.

Example 5.16. In Example 5.5, Let subset $A = \{c\}$ is GB_{s^*}-set but not α^*-set and let $A = \{a, b\}$ is a α^*-set but not GB_{s^*}-set.

The above discussions are summarized in the following diagram

Remark 5.17. The above discussions are summarized in the following diagram.



Proposition 5.18. *Let A and B are GB-sets in X. Then $A \cap B$ is a GB-set in X.*

Proof. Since A, B are GB-sets. Let $A = L_1 \cap M_1, B = L_2 \cap M_2$ where L_1, L_2 are $\theta g^* s$-open sets and M_1, M_2 are t-sets. Since intersection of two $\theta g^* s$-open sets is $\theta g^* s$-open sets and intersection of t-sets is t-set it follows that $A \cap B$ is a GB-set in X.

Remark 5.19. (1) The union of any two GB_s-sets need not be a GB_s-set.
(2) Complement of a GB_s-sets need not be a GB_s-set.

Example 5.20. In Example 5.5,
(1) A = {a} and B = {c} are GB_s-sets but $A \cup B$ = {a, c} is not a GB_s-set.
(2) $X - \{a\}$ = {b, c} is not a GB_s-set.

Remark 5.21. Let A and B be GB_s-sets in X. Then $A \cap B$ is also a GB_s-sets.

Remark 5.22. The union of two GB_{s^*}-sets is also a GB_{s^*}-sets and the complement of a GB_{s^*}-set need not be a GB_{s^*}-sets follows from the following example.

Example 5.23. In Example 5.9, Let A = {a} and B = {b} are GB_{s^*}-sets and $A \cup B$ = {a, b} is also a GB_{s^*}-sets. But $X - \{a\}$ = {b, c} is not a GB_{s^*}-set.

Proposition 5.24. *If S is a $\theta g^* s$-closed set, then*
(1) S is a GB-set.
(2) S is a GB_s-set.

Remark 5.25. The converse of the above proposition need not be true as seen from the following example.

Example 5.26. In Example 5.5, Let subset A = {c} is GB-set and GB_s-set but not a $\theta g^* s$-closed set.

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