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Polynomial-time algorithm for linear programming based on a kernel function with hyperbolic-logarithmic barrier term

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Abstract In this work, we present an interior point algorithm for linear optimization problems based on a kernel function which has a hyperbolic-logarithmic function in its barrier term. This kernel function was first proposed by the authors themselves for semi-definite programming (SDP) problems in [18]. By simple analysis tools, several properties of the proposed kernel function are used to compute the total number of iterations. We show that the worst-case iteration complexity of our algorithm for large-update methods improves the obtained iteration bounds based on the first trigonometric [5] as well as the classic kernel functions. For small-update methods, we derive the best known iteration bound. Numerical tests reveal that the proposed kernel function has promising results comparing with some existing kernel functions.

1 Introduction

After the landmark paper of Karmarkar [9], linear programming (LP) revitalized as an active area of research. The interior-point methods (IPMs) provide a powerful tool for solving optimization problems and are now among the most efficient methods from computational point of view.
In this paper, we are interested to solve linear programming problems (LP) by one of IPMs, which is the primal-dual central trajectory method. Most IPMs for LP are based on the logarithmic barrier function [6, 16] with complexity $\mathcal{O}(n \ln \frac{n}{\epsilon})$, for large-update methods, where n is the size of the problem and ϵ is the accuracy parameter.

Peng et al. in [12, 13, 14] were the first to analyse primal-dual IPMs for LP based on a class of barrier functions that is defined by the so-called *self-regular kernel functions*. They improved the theoritical complexity bound for large-update IPMs from $\mathcal{O}(n \ln \frac{n}{\epsilon})$ to the currently best known iteration bound for these types of methods, namely, $\mathcal{O}(\sqrt{n} \ln(n) \ln \frac{n}{\epsilon})$.

In 2004, Bai et al [1] proposed primal-dual IPMs for LP based on the so-called *eligible kernel functions* which are not necessarily self-regular. Since then, several kernel functions have been introduced. For instance, we refer to [3, 4, 7, 17, 11, 8] for recent works in this field.

Very recently, Touil and Chikouche [18, 19] introduced a new type of kernel functions for SDP, neither trigonometric nor exponential. The proposed kernel function in [18] has the following expression

$$\psi(t) = \left(1 + \frac{2\coth(1)}{\sinh^2(1)}\right)\frac{t^2-1}{2} + \coth^2(t) - \coth^2(1) - \log t, \ \forall t > 0. \tag{1.1}$$

The aim of this paper is to study a large-update primal-dual interior-point algorithm for solving LO problems based on this kernel function.

The paper is organized as follows: In section 2, we describe the linear programming problem to be studied, thereafter we give briefly the central trajectory method based on kernel functions. The generic interior point algorithm of this method is presented in the last of this section. In section 3, we present our kernel function and its properties to determine an effective search direction. In section 4, we analyze the complexity bound of the algorithm for both large- and small-update methods. In section 5, we present numerical experiments to illustrate the effectiveness of our kernel function in comparison with some existing kernel functions.

Throughout the paper, we use the following notations. $\mathbb{R}^n, \mathbb{R}^n_+$ and $\mathbb{R}^n_{++}$ denote the set of : n-dimensional, nonnegative and positive vectors, respectively. For $x, s \in \mathbb{R}^n$, xs and $\frac{x}{s}$ denote the component-wise product and division of the vectors x and s, respectively. For $x \in \mathbb{R}^n$, we denote $\|x\|$ and $X = diag(x)$ by the 2-norm of the vector x and the n diagonal matrix with the components of the vector x, the diagonal entries, respectively. e denotes the n-dimensional vector of ones. Finally, if $f(x) \geq 0$ is a real valued function of a real nonnegative variable, we write $f(x) = \mathcal{O}(g(x))$ if $f(x) \leq cg(x)$ for some positive constant c, and $f(x) = \Theta(g(x))$ if $c_1 x \leq f(x) \leq c_2 x$ for two positive constants c_1 and c_2.

2 The central trajectory via kernel functions

We consider the primal LP problem (P) in the standard form

$$(P)\begin{cases} \min c^T x \\ Ax = b, \\ x \geq 0, \end{cases}$$

where $A \in \mathbb{R}^{m\times n}$, $c \in \mathbb{R}^n$ and $b \in \mathbb{R}^m$ are given and $x \in \mathbb{R}^n$ is the vector of variables, and its dual problem

$$(D)\begin{cases} \max b^T y \\ A^T y + s = c, \\ s \geq 0, \end{cases}$$

where $y \in \mathbb{R}^m$ and $s \in \mathbb{R}^n$ are the vectors of variables.
Throughout of this paper, we assume that:
(H_1) : The matrix A has full ranked, i.e., rank$(A) = m \leq n$.
(H_2) : (P) and (D) satisfy the interior-point condition (IPC), i.e., there exists (x^0, y^0, s^0) such that

$$Ax^0 = b,\ x^0 > 0,\ \ A^T y^0 + s^0 = c,\ s^0 > 0.$$

To study (P), we replace it by the perturbed equivalent problem

$$(P)_\mu\begin{cases} \min c^T x - \mu \sum_{i=1}^{n} \ln x_i \\ Ax = b, \\ x > 0,\ \mu > 0. \end{cases}$$

We can also study (D) according to its perturbed dual

$$(D)_\mu\begin{cases} \max b^T y + \mu \sum_{i=1}^{n} \ln s_i \\ A^T y + s = c, \\ s > 0,\ \mu > 0. \end{cases}$$

The main advantage of $(P)_\mu$ resides in its strict convexity, as a consequence the conditions of optimality are necessary and sufficient. Thus, finding the optimal solutions of (P) and (D) is equivalent to solve the following nonlinear system by applying **KKT** to $(P)_\mu$ and $(D)_\mu$

$$\begin{cases} Ax = b,\ x > 0, \\ A^T y + s = c, s > 0, \\ xs = \mu e,\ \mu > 0. \end{cases} \tag{2.1}$$

Due to assumptions (H_1) and (H_2), system (2.1) has a unique solution for each $\mu > 0$, denoted (x_μ, y_μ, s_μ).
The set of all solutions is called the $\mu-$center (or the central path) of (P) and (D). It has been shown that when μ tends to zero, the limit of the central path exists and converges to the optimal solutions of (P) and (D) [1].

Now, applying Newton's method to system (2.1) for computing the search direction $(\Delta x, \Delta y, \Delta s)$, leads to the following linear system

$$\begin{cases} A\Delta x = 0, \\ A^T\Delta y + \Delta s = 0, \\ s\Delta x + x\Delta s = \mu e - xs. \end{cases} \tag{2.2}$$

Let us define the scaled vector v and the scaled search directions vectors d_x and d_s as follows

$$v = \sqrt{\frac{xs}{\mu}}, \quad d_x = \frac{v\Delta x}{x}, \quad d_s = \frac{v\Delta s}{s}. \tag{2.3}$$

Using (2.3) and by simple calculations, system (2.2) is converted to

$$\begin{cases} \overline{A}d_x = 0, \\ \overline{A}^T\Delta y + d_s = 0, \\ d_x + d_s = v^{-1} - v = -\nabla\Psi_c(v), \end{cases} \tag{2.4}$$

where $\overline{A} = \frac{1}{\mu}AV^{-1}X$, $V = \text{diag}(v)$, $X = \text{diag}(x)$, and $\Psi_c(v) = \sum_{i=1}^{n}\psi_c(v_i)$, is the proximity barrier function of the classical kernel function $\psi_c(t)$

$$\psi_c(t) = \left(\frac{t^2-1}{2} - \log t\right).$$

In this paper, we replace $\psi_c(t)$ by the kernel function defined in (1.1). Note that the triple (x, y, s) coincides with the μ-center (x_μ, y_μ, s_μ) if and only if $v = e$.
System (2.4) is transformed to

$$\begin{cases} \overline{A}d_x = 0, \\ \overline{A}^T\Delta y + d_s = 0, \\ d_x + d_s = -\nabla\Psi(v). \end{cases} \tag{2.5}$$

Since $\text{rank}(A) = m$, then system (2.5) has a unique solution $(d_x, \Delta y, d_s)$. Having d_x and d_s we can obtain Δx and Δs.
The Newton iterate with step size α is constructed according to

$$x_+ = x + \alpha\Delta x, y_+ = y + \alpha\Delta y, s_+ = s + \alpha\Delta s, \tag{2.6}$$

where the step size $\alpha \in]0, 1]$ and satisfies $(x_+, s_+) > 0$.
Now, we can define the norm-based proximity measure $\sigma(v)$ as

$$\sigma(v) = \frac{1}{2}\left\|d_x + d_s\right\|. \tag{2.7}$$

The generic IPM outlined above can be summarized in the following algorithm.

Primal-dual algorithm for LP
Input
A threshold parameter $\tau \geq 1$;
an accuracy parameter $\epsilon > 0$;
a fixed barrier update parameter $\theta \in]0, 1[$;
(x^0, y^0, s^0) is a strictly feasible point and $\mu^0 = 1$
such that $\Psi(v^0) \leq \tau$.
begin algorithm
$x := x^0; s := s^0; \mu := \mu^0;$
While $n\mu \geq \epsilon$
begin
$\mu := (1-\theta)\mu;$
While $\Psi(v) > \tau$
begin
Solve system (2.5) and choose a suitable step size α;
$x := x + \alpha\Delta x;$
$y := y + \alpha\Delta y;$
$s := s + \alpha\Delta s;$
$v := \sqrt{\frac{xs}{\mu}};$
end
end
end algorithm

3 The kernel function and its properties

In this section, we present some technical lemmas of the kernel function defined in (1.1). For the proves, we refer the reader to [18].

Lemma 3.1. *(Lemma 3.2 in [18]) we have on the interval* $]0, +\infty[$

(i) ψ *is convex exponentially ; i.e.,*

$$\psi(\sqrt{t_1 t_2}) \leq \frac{1}{2}(\psi(t_1) + \psi(t_2)), \ \forall t_1, t_2 > 0.$$

(ii) $t\psi''(t) - \psi'(t) > 0.$

(iii) ψ'' *is monotonically decreasing.*

Lemma 3.2. *(Lemmas 3.3 and 3.7 in [18]) For* $\psi(t)$, *we have*

(i) $2(t-1)^2 \leq \psi(t) \leq \left(\frac{\psi'(t)}{2}\right)^2, \ \forall t > 0.$

(ii) $\psi(t) \leq \frac{1}{2}\psi''(1)(t-1)^2, \ \forall t \geq 1.$

(iii) $\sigma^2(v) \geq \Psi(v).$

Lemma 3.3. *(Lemma 3.4 in [18]) Let* $\varphi : [0, +\infty[\to [1, +\infty[$ *be the inverse function of* $\psi(t)$ *for* $t \geq 1$ *and* $\rho : [0, +\infty[\to]0, 1]$ *be the inverse function of* $-\frac{1}{2}\psi'(t)$ *for* $0 < t \leq 1$, *then*

(i) $1 + \sqrt{\frac{2z}{\psi''(1)}} \leq \varphi(z) \leq 1 + \sqrt{\frac{z}{2}}, \forall z \in [0, +\infty[.$

(ii) $\coth t \leq \sqrt{2}\,(z+1)^{\frac{1}{3}}$, $z = -\dfrac{1}{2}\psi'(t) \geq 0, \forall t \in]0,1]$.

Now, we derive an estimate for the effect of updating the barrier parameter μ on the value of the proximity function during an iteration.

Theorem 3.4 ([1]). *For any $v > 0$ and $\beta > 1$, we have*

$$\Psi(\beta v) \leq n\psi\left(\beta\varphi\left(\frac{\Psi(v)}{n}\right)\right).$$

Corollary 3.5. *(Corollary 3.6 in [18]) Let θ be such that $0 < \theta < 1$. If $\Psi(v) \leq \tau$, then*

$$\Psi(\beta v) \leq \frac{\psi''(1)}{4(1-\theta)}\left(\theta\sqrt{2n} + \sqrt{\tau}\right)^2 =: \Psi_0, \text{ with } \beta = \frac{1}{\sqrt{1-\theta}} > 1, \tag{3.1}$$

where, Ψ_0 is an upper bound of $\Psi(\beta v)$, during the process of the algorithm.

4 Complexity analysis of the algorithm

The aim of this section is to compute the value of the step size α such that the new iterate (x_+, y_+, s_+) is strictly feasible and the proximity function $\Psi(v)$ is decreasing.
From (2.6) and by using (2.3), we have

$$x_+ = \frac{x}{v}(v + \alpha d_x), \quad s_+ = \frac{s}{v}(v + \alpha d_s).$$

It follows that the new v-vector is given by

$$v_+ = \sqrt{\frac{x_+ s_+}{\mu}} = \sqrt{(v + \alpha d_x)(v + \alpha d_s)}.$$

From (i) of Lemma 3.1, we have

$$\Psi(v_+) \leq \frac{1}{2}\left(\Psi(v + \alpha d_x) + \Psi(v + \alpha d_s)\right).$$

For fixed μ, let us define the difference of proximities between a new iterate and a current iterate as

$$f(\alpha) = \Psi(v_+) - \Psi(v).$$

In the remainder of this section, we put for simplicity $\sigma := \sigma(v)$.

Lemma 4.1. *(Lemmas 4.4 and 4.7 in [18]) The largest possible value of the step size α^* is*

$$\alpha^* = \frac{\rho(\sigma) - \rho(2\sigma)}{2\sigma}.$$

Furthermore

$$\alpha^* \geq \frac{1}{\psi''(\rho(2\sigma))},$$

and we have for all $\alpha \in [0, \alpha^*]$

$$f(\alpha) \leq -\alpha\sigma^2.$$

The next corollary present the decrease of the proximity function in the inner iteration.

Corollary 4.2. *(Lemma 4.5 in [1] and Theorem 4.8 in [18]) Let us set $\bar{\alpha} = \dfrac{1}{\psi''(\rho(2\sigma))}$, as the default step size. Suppose that $\sigma \geq 1$, we have*

$$f(\bar{\alpha}) \leq -\frac{\sigma^2}{\psi''(\rho(2\sigma))} \leq -\frac{\Psi(v)^{\frac{1}{3}}}{130}. \tag{4.1}$$

We need to compute how many inner iterations are required to return to the situation where $\Psi(v) \leq \tau$ after $\mu-$update. Let us define the value of $\Psi(v)$ after $\mu-$update as Ψ_0, and the subsequent values in the same outer iteration as $\Psi_k, k = 1, ..., K$, where K stands for the total number of inner iterations in the outer iteration. By the definition of $f(\alpha)$ and according to (4.1) , for $k = 1, ..., K-1$, we obtain

$$\Psi_{k+1} \leq \Psi_k - \frac{\Psi_k^{\frac{1}{3}}}{130}.$$

By taking $t_k = \Psi_k$, $\beta = \frac{1}{130}$ and $\gamma = \frac{2}{3}$, we can get the following lemma.

Lemma 4.3. *(Lemma 4.9 in [18]) Let K be the total number of inner iterations in the outer iteration. Then, we have*

$$K \leq \left\lceil \frac{t_0^{\gamma}}{\beta\gamma} \right\rceil = 195\,\Psi_0^{\frac{2}{3}},$$

where Ψ_0 is an upper bound of $\Psi(\beta v)$, during the process of the algorithm.

Now, we derive the complexity bounds for large and small-update methods.

Theorem 4.4. *(Theorem 4.10 in [18]) The total number of iterations to obtain an approximate solution is bounded by*

$$\left(195\,\Psi_0^{\frac{2}{3}}\right)\left(\frac{\log\frac{n}{\epsilon}}{\theta}\right). \tag{4.2}$$

Proof. We known that the number of outer iterations for the situation $n\mu \leq \epsilon$ is bounded by $\frac{1}{\theta}(\log\frac{n}{\epsilon})$. Knowing that an upper bound for the total number of iterations is obtained by multiplying the number of inner and outer iterations, we obtain the result thanks to the above lemma. □

If $\tau = \mathcal{O}(n)$ and $\theta = \Theta(1)$, we have $\mathcal{O}\left(n^{\frac{2}{3}}\log\frac{n}{\epsilon}\right)$ iterations for large-update IPMs.
For small-update IPMs with $\tau = \mathcal{O}(1)$ and $\theta = \Theta\left(\frac{1}{\sqrt{n}}\right)$, we get $\mathcal{O}\left(\sqrt{n}\log\frac{n}{\epsilon}\right)$ iterations.

5 Numerical tests

In this section, we carry out numerical experiments of the interior point algorithm based on the kernel functions given in **Table** 1. These functions differs by the type of their barrier term, furthermore, their complexity bounds is greater or equal to the complexity of our kernel function.
Our experiment are implemented in MATLAB R2012b and performed on Supermicro dual-2.80 GHz Intel Core i5 server with 4.00 Go RAM. We have taken $\epsilon = 10^{-8}$, $\tau = n$ and $\theta \in \{0.9, 0.99\}$.
We chose a practical step size α as in [10] i.e., $\alpha = \min(\alpha_x, \alpha_s)$, with

$$\alpha_x = \begin{cases} \min\limits_{i\in I_0}(-\dfrac{x_i}{\Delta x_i}) & \text{if } I_0 \neq \emptyset, \\ 1 & \text{elsewhere,} \end{cases}$$

where

$$I_0 = \{i \in \{1, ..., n\} : \Delta x_i < 0\}.$$

And

$$\alpha_s = \begin{cases} \min\limits_{i\in I_1}(-\dfrac{s_i}{\Delta s_i}) & \text{if } I_1 \neq \emptyset, \\ 1 & \text{elsewhere,} \end{cases}$$

where

$$I_1 = \{i \in \{1, ..., n\} : \Delta s_i < 0\}.$$

Table 1. Proposed kernel functions.

Kernel function ψ_i	Choice of p	Complexity of large-u
$\psi_1(t) = \frac{t^2-1}{2} - \int_1^t e^{p(\frac{1}{i}-1)},\ p \geq 1$	$p = log(1+n)$	$\mathcal{O}\left(\sqrt{n}\log n \log \frac{n}{\epsilon}\right)$
$\psi_2(t) = \frac{t^2-1}{2} - \int_1^t \left[\frac{\sec \frac{\pi x}{2x+2}}{\sqrt{2}}\right]^{3p},\ p \geq 2$	$p = \frac{log(n)}{2}$	$\mathcal{O}\left((1+2k)\sqrt{n}\log n \right.$ l
$\psi_3(t) = \frac{t^2-1-\log t}{2} + \frac{t^{1-p}-1}{2(p-1)},\ p \geq 2$	$p = \frac{log(n)}{2}$	$\mathcal{O}\left(\sqrt{n}\log n \log \frac{n}{\epsilon}\right)$
$\psi_4(t) = \frac{t^2-1}{2} - \log t + \frac{1}{8}\tan^2\left(\frac{1-t}{2+4t}\pi\right)$	-	$\mathcal{O}\left(n^{\frac{2}{3}}\log \frac{n}{\epsilon}\right)$
$\psi_5(t) = t^2 + \frac{t^{1-q}}{q-1} - \frac{q}{q-1} + \frac{4}{\pi p}\left[\tan^p\left(\frac{\pi}{2t+2}\right) - 1\right],\ p \geq 2,\ q > 1$	$p = q = log(n)$	$\mathcal{O}\left(\sqrt{n}\log n \log \frac{n}{\epsilon}\right)$
$\psi_{\text{new}}(t) = \left(1 + \frac{2\coth(1)}{\sinh^2(1)}\right)\frac{t^2-1}{2} + \coth^2(t) - \coth^2(1) - \log t$	-	$\mathcal{O}\left(n^{\frac{2}{3}}\log \frac{n}{\epsilon}\right)$

Example 5.1 ([3]). The matrix A is defined as

$$A(i,j) = \begin{cases} 1 & \text{if } i = j \text{ or } j = i+m, \\ 0 & \text{elsewhere,} \end{cases}$$

$$c(i) = -1,\ c(i+m) = 0 \text{ and } b(i) = 2, \quad \text{for } i = 1, ..., m.$$

We start by an initial point (x^0, y^0, s^0) such that

$$x^0 = e,\ y^0(i) = -2 \text{ and } s^0(i) = 1,\ s^0(i+m) = 2, \text{ for } i = 1, ..., m.$$

The obtained optimal solutions for $n = 2m$ where $m \in \{5, 25, 50, 100, 200, 400, 1000\}$ are

$$x^*(i) = 2,\ x^*(i+m) = 0,\ y^*(i) = -1,\ s^*(i) = 0, \text{ and } s^*(i+m) = 1, \text{ for } i = 1, ..., m.$$

We present in the following table the number of iterations necessary to obtain the optimal solution corresponding to each function given in **Table** 1. m represents the size of the example and we use **bold** font to highlight the best, i.e., the smallest, iteration number.

Table 2. Number of iterations for Example 1 with different sizes $n = 2m$.

θ	m	ψ_1	ψ_2	ψ_3	ψ_4	ψ_{new}	$\psi_{5,p,q}$	
							$p=4,\ q=6$	$p=4,\ q=\log(n)$
$\theta = 0.9$	5	21	58	25	16	**12**	44	33
	25	23	63	15	17	**13**	52	19
	50	23	63	15	17	**13**	52	**13**
	100	24	69	16	19	**14**	60	33
	200	24	69	15	19	**14**	60	36
	400	24	69	16	19	**14**	60	54
	1000	25	69	17	20	**15**	61	46
$\theta = 0.99$	5	**21**	89	**21**	28	**21**	27	75
	25	**21**	89	30	28	**21**	27	57
	50	24	104	29	31	**23**	32	88
	100	24	104	27	31	23	32	**17**
	200	24	104	30	31	**23**	32	36
	400	24	104	24	31	**23**	32	53
	1000	24	104	31	31	**23**	32	57

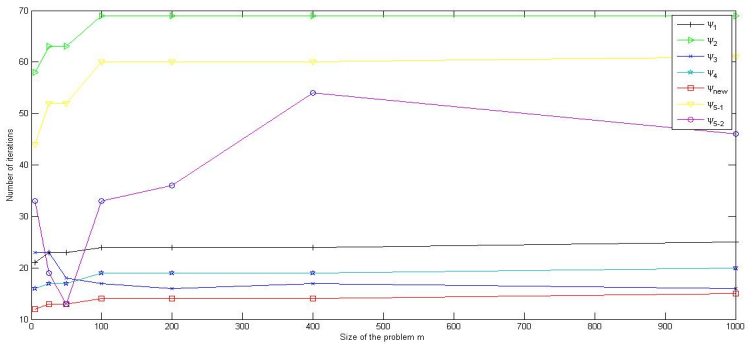

Figure 1. Number of iterations until duality gap below to 10^{-8} for $\theta = 0.9$.

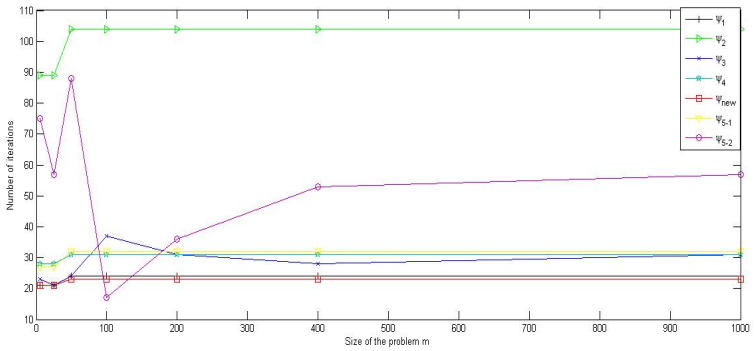

Figure 2. Number of iterations until duality gap below to 10^{-8} for $\theta = 0.99$.

5.1 Comments

From **Table** 2, we conclude that:
• For both values of θ, the algorithm based on each of the tested kernel functions converges to the optimal solution of Example 1.
• Numerical tests prove the efficiency of our kernel function since the best iteration complexity was achieved in all experiments.
In figures 1 and 2, we plot the number of iterations taken by each kernel function to obtain the optimal solution below 10^{-8} in terms of the dimension m for $\theta = 0.9$ and $\theta = 0.99$, respectively.

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