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Dual focal curves of dual spacelike curves in the dual Lorentzian $\mathbb{D}_1^3$

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Abstract. In this paper, we study dual focal curves of spacelike curves with timelike principal normal in the Dual Lorentzian 3-space $\mathbb{D}_1^3$. We characterize dual focal curves in terms of their dual focal curvatures.

1 Introduction

Dual numbers were introduced by W. K. Clifford (1849-79) as a tool for his geometrical investigations. After him E. Study used dual numbers and dual vectors in his research on the geometry of lines and kinematics. He devoted special attention to the representation of directed lines by dual unit vectors and defined the mapping that is known by his name.

In this paper, we study dual focal curves of spacelike curves with timelike principal normal in the Dual Lorentzian 3-space $\mathbb{D}_1^3$. We characterize dual focal curves in terms of their dual focal curvatures.

2 Dual Focal Of Dual Spacelike Curves With Timelike Principal Normal According To Dual Frenet Frame In $\mathbb{D}_1^3$

Let $\left\{\hat{\mathbf{T}}, \hat{\mathbf{N}}, \hat{\mathbf{B}}\right\}$ be the dual Frenet frame of the differentiable dual spacelike curve in the dual space $\mathbb{D}_1^3$. Then the dual Frenet frame equations are

$$\begin{aligned} \hat{\mathbf{T}}' &= \kappa\mathbf{N} + \varepsilon(\kappa^*\mathbf{N} + \kappa\mathbf{N}^*), \\ \mathbf{N}' &= \kappa\mathbf{T} + \tau\mathbf{B} + \varepsilon\left(\kappa^*\mathbf{T} + \kappa\mathbf{T}^* + \tau^*\mathbf{B} + \tau\mathbf{B}^*\right), \\ \hat{\mathbf{B}}' &= \tau\mathbf{N} + \varepsilon\left(\tau^*\mathbf{N} + \tau\mathbf{N}^*\right), \end{aligned}$$

where $\hat{\kappa} = \kappa + \varepsilon\kappa^*$ is nowhere pure dual natural curvatures and $\hat{\tau} = \tau + \varepsilon\tau^*$ is nowhere pure dual torsion.

Denoting the dual focal curve by $\hat{\wp}$ we can write

$$\hat{\wp}(s) = (\hat{\gamma} + \hat{\mathfrak{m}}_1\hat{\mathbf{N}} + \hat{\mathfrak{m}}_2\hat{\mathbf{B}})(s), \tag{2.1}$$

where the coefficients $\hat{\mathfrak{m}}_1$, $\hat{\mathfrak{m}}_2$ are smooth functions of the parameter of the curve $\hat{\gamma}$, called the first and second dual focal curvatures of $\hat{\gamma}$, respectively.

The formula (2.1) is separed into the real and dual part, we have

$$\wp(s) = (\gamma + \mathfrak{m}_1\mathbf{N} + \mathfrak{m}_2\mathbf{B})(s), \tag{2.2}$$

$$\wp^*(s) = \left(\gamma^* + \mathfrak{m}_1\mathbf{N}^* + \mathfrak{m}_1^*\mathbf{N} + \mathfrak{m}_2\mathbf{B}^* + \mathfrak{m}_2^*\mathbf{B}\right)(s).$$

Theorem 2.1. *Let $\hat{\gamma} : I \longrightarrow \mathbb{D}_1^3$ be a unit speed dual spacelike curve and $\hat{\wp}$ its dual focal curve on $\mathbb{D}_1^3$. Then,*

$$\wp = \gamma - \frac{1}{\kappa}\mathbf{N} - \frac{\kappa'}{\kappa^2\tau}\mathbf{B}, \tag{2.3}$$

$$\wp^* = \gamma^* - \frac{1}{\kappa}\mathbf{N}^* + \frac{\kappa^*}{\kappa^2}\mathbf{N} - \frac{\kappa'}{\kappa^2\tau}\mathbf{B}^* + \left(\frac{-(\kappa^*)'\kappa^2 + 2\kappa\kappa^*\kappa'}{\kappa^4\tau} + \frac{\tau^*\kappa'}{\kappa^2\tau^2}\right)\mathbf{B}. \tag{2.4}$$

Proof. Assume that $\hat{\gamma}$ is a unit speed dual spacelike curve and $\hat{\wp}$ its dual focal curve on $\mathbb{D}^3_1$. So, by differentiating of the formula (2.1), we get

$$\hat{\wp}(s)' = (1 + \hat{\kappa}\hat{\mathfrak{m}}_1)\hat{\mathbf{T}} + (\hat{\mathfrak{m}}_1' + \hat{\tau}\hat{\mathfrak{m}}_2)\hat{\mathbf{N}} + (\hat{\tau}\hat{\mathfrak{m}}_1 + \hat{\mathfrak{m}}_2')\hat{\mathbf{B}}. \tag{2.5}$$

Using above equation, the first 2 components vanish, we have

$$\begin{aligned} \kappa\mathfrak{m}_1 &= -1, \\ \kappa\mathfrak{m}_1^* + \kappa_1^*\mathfrak{m} &= 0, \\ \mathfrak{m}_1' + \tau\mathfrak{m}_2 &= 0, \\ (\mathfrak{m}_1^*)' + \tau\mathfrak{m}_2^* + \tau^*\mathfrak{m}_2 &= 0. \end{aligned}$$

Considering equations above system, we have

$$\begin{aligned} \mathfrak{m}_1 &= -\frac{1}{\kappa}, \\ \mathfrak{m}_1^* &= \frac{\kappa^*}{\kappa^2}, \\ \mathfrak{m}_2 &= -\frac{\kappa'}{\kappa^2\tau}, \\ \mathfrak{m}_2^* &= \frac{-(\kappa^*)'\kappa^2 + 2\kappa\kappa^*\kappa'}{\kappa^4\tau} + \frac{\tau^*\kappa'}{\kappa^2\tau^2}. \end{aligned}$$

By means of obtained equations, we express (2.3) and (2.4). This completes the proof.

Corollary 2.2. *Let $\hat{\gamma} : I \longrightarrow \mathbb{D}^3_1$ be a unit speed dual spacelike curve and $\hat{\wp}$ its dual focal curve on $\mathbb{D}^3_1$. Then, the dual focal curvatures of $\hat{\wp}$ are*

$$\begin{aligned} \mathfrak{m}_1 &= -\frac{1}{\kappa}, \\ \mathfrak{m}_1^* &= \frac{\kappa^*}{\kappa^2}, \\ \mathfrak{m}_2 &= -\frac{\kappa'}{\kappa^2\tau}, \\ \mathfrak{m}_2^* &= \frac{-(\kappa^*)'\kappa^2 + 2\kappa\kappa^*\kappa'}{\kappa^4\tau} + \frac{\tau^*\kappa'}{\kappa^2\tau^2}. \end{aligned}$$

In the light of Theorem 2.1, we express the following corollary without proof:

Corollary 2.3. *Let $\hat{\gamma} : I \longrightarrow \mathbb{D}^3_1$ be a unit speed dual spacelike curve and $\hat{\wp}$ its dual focal curve on $\mathbb{D}^3_1$. If $\hat{\kappa}$ and $\hat{\tau}$ are constant then, the dual focal curvatures of $\hat{\wp}$ are*

$$\begin{aligned} \mathfrak{m}_1 &= \textit{constant} \neq 0, \\ \mathfrak{m}_1^* &= \textit{constant} \neq 0, \\ \mathfrak{m}_2 &= 0, \\ \mathfrak{m}_2^* &= 0. \end{aligned}$$

Corollary 2.4. *Let $\hat{\gamma} : I \longrightarrow \mathbb{D}^3_1$ be a unit speed dual spacelike helix and $\hat{\wp}$ its dual focal curve on $\mathbb{D}^3_1$. Then,*

$$\hat{\wp}(s) = \hat{\gamma}(s) + \hat{\mathfrak{m}}_1\hat{\mathbf{N}}(s),$$

where $\hat{\mathfrak{m}}_1$ is first dual focal curvatures of $\hat{\gamma}$.

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