Palestine Journal of Mathematics

Vol. 4(1) (2015) , 189–197

A note on difference cordial graphs

R. Ponraj, S. Sathish Narayanan and R. Kala

Communicated by Ayman Badawi

MSC 2010 Classifications: 05C78.

Keywords and phrases: Lotus inside a circle, Pyramid, Permutation graphs, t-fold wheel, corona.

Abstract Let G be a (p,q) graph. Let $f : V(G) \to \{1,2,\dots,p\}$ be a function. For each edge uv, assign the label $|f(u) - f(v)|$. f is called a difference cordial labeling if f is a one to one map and $|e_f(0) - e_f(1)| \le 1$ where $e_f(1)$ and $e_f(0)$ denote the number of edges labeled with 1 and not labeled with 1 respectively. A graph which admits a difference cordial labeling is called a difference cordial graph. In this paper we investigate the difference cordial labeling behaviour of $K_2 + mK_1$, $K_n^c + 2K_2$, Sunflower grpah, Lotus inside a circle, Pyramid, Permutation graphs.

1 Introduction

Let $G = (V,E)$ be (p,q) graph. In this paper we have considered only simple and undirected graphs. The number of vertices of G is called the order of G and the number of edges of G is called the size G. Labeled graphs are used in several areas of science and technology such as astronomy, radar, circuit design and database management[2]. The origin of graph labeling is Graceful labeling which was introduced by Rosa [11] in the year 1967. In 1980, Cahit [1] introduced the cordial labeling of graphs. Cordiality behavior of numerous graphs were studied by several authors [4, 12, 14, 20, 5, 15, 16, 17, 18, 19, 13]. In this approach, R. Ponraj, S. Sathish Narayanan and R. Kala introduced difference cordial labeling in [6]. In [6, 7, 8, 9] difference cordial labeling behavior of several graphs like path, cycle, complete graph, complete bipartite graph, bistar, wheel, web and some more standard graphs have been investigated. In this paper, we investigate the difference cordial labeling behavior of $K_2 + mK_1$, $K_n^c + 2K_2$, Sunflower graph, Lotus inside a circle, Pyramid, Permutation graphs, book with n pentagonal pages, t-fold wheel, double fan. Let x be any real number. Then the symbol $\lfloor x \rfloor$ stands for the largest integer less than or equal to x and $\lceil x \rceil$ stands for the smallest integer greater than or equal to x. Terms and definitions not defined here are used in the sense of Harary [3].

2 Difference cordial labeling

Definition 2.1. Let G be a (p,q) graph. Let f be a map from $V(G)$ to $\{1,2,\dots,p\}$. For each edge uv, assign the label $|f(u) - f(v)|$. f is called difference cordial labeling if f is $1-1$ and $|e_f(0) - e_f(1)| \le 1$ where $e_f(1)$ and $e_f(0)$ denote the number of edges labeled with 1 and not labeled with 1 respectively. A graph with a difference cordial labeling is called a difference cordial graph.

The following results (theorem 2.1 to 2.9) are used in the subsequent section.

Theorem 2.2. *[6] Any Path is a difference cordial graph.*

Theorem 2.3. *[6] Any Cycle is a difference cordial graph.*

Theorem 2.4. *[6] If G is a (p,q) difference cordial graph, then $q \le 2p - 1$.*

Theorem 2.5. *[6] K_n is difference cordial iff $n \le 4$.*

Theorem 2.6. *[6] $K_{2,n}$ is difference cordial iff $n \le 4$.*

Theorem 2.7. *[6] $K_{3,n}$ is difference cordial iff $n \le 4$.*

Theorem 2.8. *[6] The wheel W_n is difference cordial.*

Theorem 2.9. *[10] The ladder L_n is difference cordial.*

Theorem 2.10. *[10] The prism $C_n \times P_2$ is difference cordial.*

The join of two graphs G_1 and G_2 is denoted by G_1+G_2 and whose vertex set is $V(G_1+G_2) = V(G_1) \cup V(G_2)$ and edge set $E(G_1 + G_2) = E(G_1) \cup E(G_2) \cup \{uv : u \in V(G_1), v \in V(G_2)\}$

Theorem 2.11. *Let G be a (p, q) graph. If $q > p + 1$, then $G + K_1$ is not difference cordial.*

Proof. Obviously, $G + K_1$ is a $(p + 1, p + q)$ graph. Suppose $G + K_1$ is difference cordial, then by theorem 2.4, $p + q \leq 2(p + 1) - 1$. This implies $q \leq p + 1$, a contradiction. □

Theorem 2.12. *Let G_1 be a (p_1, q_1) connected graph and G_2 be a (p_2, q_2) connected graph with $G_1 \neq K_1$ and $p_2 > 3$ then $G_1 + G_2$ is not difference cordial.*

Proof. The order and size of $G_1 + G_2$ are $p_1 + p_2$ and $q_1 + q_2 + p_1p_2$ respectively. Suppose $G_1 + G_2$ is difference cordial with $G_1 \neq K_1$ and $p_2 > 3$, then by theorem 2.4, $q_1 + q_2 + p_1p_2 \leq 2(p_1 + p_2) - 1$, $\Rightarrow p_1p_2 - p_2 - p_1 + 1 \leq 2$, $\Rightarrow 2 \geq p_2(p_1 - 1) - p_1 + 1 > 3(p_1 - 1) - p_1 + 1 > 2$, a contradiction. □

Theorem 2.13. *Let G be a (p, q) graph. Then $G + G$ is difference cordial iff $p \leq 3$ and $q \leq 1$.*

Proof. The number of vertices and edges in $G + G$ are $2p$ and $2q + p^2$ respectively. Suppose f is a difference cordial labeling of $G + G$, then by theorem 2.4, $2q + p^2 \leq 2(2p) - 1$. This implies $p \leq 3$. It follows that $q \leq 1$. When $p = 3$ and $q = 1$, the difference cordial labeling of $G + G$ is shown in figure 1.

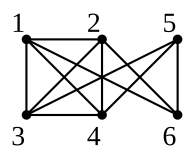

Figure 1

When $p = 3$ and $q = 0$, $G + G \cong K_{3,3}$ which is difference cordial by theorem 2.7. When $p = 2$ and $q = 1$, $G + G \cong K_4$ which is difference cordial by theorem 2.5. When $p = 2$ and $q = 0$, $G + G \cong K_{2,2}$ which is difference cordial by theorem 2.6. When $p = 1$, q must be 0. Here, $G + G \cong P_2$ this is difference cordial by theorem 2.2. □

Theorem 2.14. *Let G be a (p, q) difference cordial graph with k $(k > 1)$ vertices of degree $p-1$. Then $p \leq 7$.*

Proof. Obviously, $e_f(1) \leq p-1$. Let u_i $(1 \leq i \leq k)$ be the vertex of G such that $deg(u_i) = p-1$ $(1 \leq i \leq k)$. Then $e_f(0) \geq (p-3)+(p-3)-1+\cdots+(p-3)-(k-1) = k(p-3) - \frac{k(k-1)}{2}$. This implies, $e_f(0) - e_f(1) \geq k(p-3) - \frac{k(k-1)}{2} - p + 1$. It follows that $p \leq 7$. □

Theorem 2.15. *$K_2 + mK_1$ is difference cordial iff $m \leq 4$.*

Proof. Suppose $K_2 + mK_1$ is difference cordial then by theorem 2.14, $m \leq 5$. Let $V(K_2 + mK_1) = \{u, v, w_i : 1 \leq i \leq m\}$ and $E(K_2 + mK_1) = \{uv, uw_i, vw_i : 1 \leq i \leq m\}$. When $m = 5$, the maximum value of $e_f(1)$ occur when $f(u) = 2$,$f(v) = 4$, $f(w_1) = 1$, $f(w_2) = 3$, $f(w_3) = 5$, $f(w_4) = 6$ and $f(w_5) = 7$. It follows that, $e_f(1) \leq 4$, $e_f(0) \geq q - 4 \geq 7$. This implies $e_f(0) - e_f(1) \geq 3$. Hence $K_2 + 5K_1$ is not difference cordial. Since, $K_2 + K_1 \cong C_3$, by theorem 2.3, $K_2 + K_1$ is difference cordial. The difference cordial labeleling of $K_2 + 2K_1$, $K_2 + 3K_1$ and $K_2 + 4K_1$ are shown in figure 2.

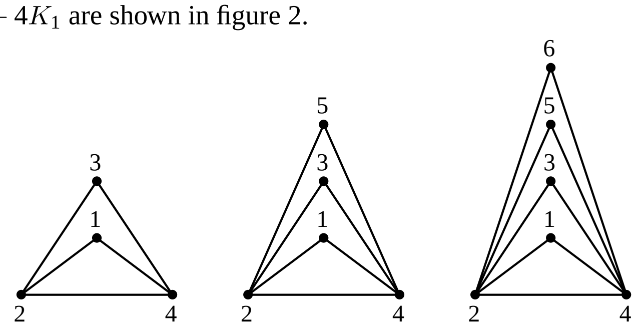

Figure 2

□

Theorem 2.16. *$K_n^c + 2K_2$ is difference cordial iff $n \leq 2$.*

Proof. Let $V(K_n^c + 2K_2) = \{u, v, w, z, u_i : 1 \leq i \leq n\}$ and $E(K_n^c + 2K_2) = \{uv, wz\} \cup \{uu_i, vu_i, wu_i, zu_i : 1 \leq i \leq n\}$. Clearly, $e_f(1) \leq n + 3$. Since the degree of the vertices u, v, w and z is $n + 1$, $e_f(0) \geq (n-1) + (n-1) - 1 + (n-1) + (n-1) - 1 \geq 4n - 6$. Hence, $e_f(0) - e_f(1) \geq 3n - 9$. This implies $n \leq 3$. Suppose $n = 3$. Here, $e_f(1) \leq 6$. Also $e_f(0) \geq q - e_f(1) \geq 8$. Therefore, $e_f(0) - e_f(1) \geq 2$. Hence, $K_3^c + +2K_2$ is not difference cordial. For $n \leq 2$, the difference cordial labeling is given in figure 3.

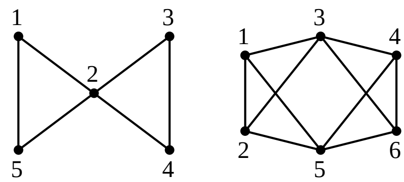

Figure 3

□

The sunflower graph S_n is obtained by taking a wheel with central vertex v_0 and the cycle $C_n : v_1v_2 \ldots v_nv_1$ and new vertices $w_1w_2 \ldots w_n$ where w_i is joined by vertices $v_i, v_{i+1(\text{mod } n)}$.

Theorem 2.17. *The sunflower graph S_n is difference cordial, for all n.*

Proof. Define $f : V(S_n) \to \{1, 2, \ldots 2n+1\}$ by $f(v_0) = 1$, $f(v_i) = 2i$, $1 \leq i \leq n$, $f(w_i) = 2i + 1$, $1 \leq i \leq n$. Now $e_f(0) = 2n$ and $e_f(1) = 2n$. Therefore f is a difference cordial labeling. □

The Lotus inside a circle LC_n is a graph obtained from the cycle $C_n : u_1u_2 \ldots u_nu_1$ and a star $K_{1,n}$ with central vertex v_0 and the end vertices $v_1v_2 \ldots v_n$ by joining each v_i to u_i and $u_{i+1(\text{mod } n)}$.

Theorem 2.18. *The Lotus inside a circle LC_n is difference cordial, for all n.*

Proof. Define a map f from the vertex set of LC_n to the set $\{1, 2, \ldots 2n+1\}$ as follows: $f(v_0) = 1$, $f(v_i) = 2i$, $1 \leq i \leq n$, $f(u_i) = 2i + 1$, $1 \leq i \leq n$. Clearly f is a difference cordial labeling. □

A Lotus inside a circle LC_4 with a difference cordial labeling is shown in figure 4.

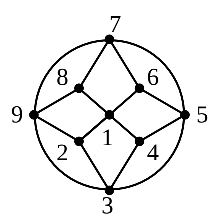

Figure 4

The graph obtained by arranging vertices into a finite number of rows with i vertices in the i^{th} row and in every row the j^{th} vertex and $j + 1^{st}$ vertex of the next row is called the Pyramid. We denote the Pyramid with n rows by Py_n.

Theorem 2.19. *All Pyramids are difference cordial.*

Proof. Let $a_{i,j}$ $(1 \leq j \leq i)$ be the vertices in the i^{th} row. Define an injective map f from the vertices of the Pyramid Py_n to the set $\left\{1, 2, 3 \ldots \frac{n(n+1)}{2}\right\}$ by $f(a_{i,j}) = \frac{1}{2}(j-1)(2n-j)+i$, $j \leq i \leq n$. Clearly, $e_f(0) = e_f(1) = \frac{n(n-1)}{2}$. Therefore, f is a difference cordial labeling of the Pyramid. □

Example 2.20. The Pyramid Py_6 with a difference cordial labeling is shown in figure 5.

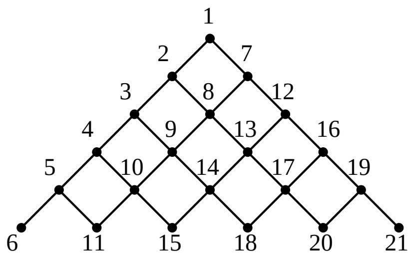

Figure 5

The graph $P_n + 2K_1$ is called a double fan DF_n.

Theorem 2.21. *The double fan DF_n is difference cordial iff $n \leq 4$.*

Proof. Note that DF_n is a $(n+2, 3n-1)$ graph. Suppose DF_n is difference cordial, then by theorem 2.4, $3n-1 \leq 2(n+2)-1$. It follows that $n \leq 4$. Since $DF_1 \cong P_3$, $DF_3 \cong w_4$, using theorem 2.2 and theorem 2.8, DF_1 and DF_3 are difference cordial. The difference cordial labeling of DF_2 and DF_4 is given in figure 6.

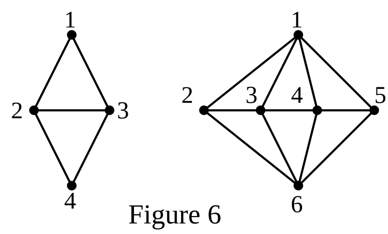

Figure 6

□

Theorem 2.22. *Books with n pentagonal pages are difference cordial.*

Proof. Let G be a book with n pentagonal pages. Let $V(G) = \{u_i, v_i, w_i : 1 \leq i \leq n\} \cup \{u, v\}$ and $E(G) = \{uu_i, u_iw_i, w_iv_i, v_iv : 1 \leq i \leq n\}$. Define a map $f : V(G) \to \{1, 2, \ldots, 3n+2\}$ by

$$\begin{aligned} f(u_i) &= 3i-2 \quad 1 \leq i \leq n \\ f(w_i) &= 3i-1 \quad 1 \leq i \leq n \\ f(v_i) &= 3i \qquad\ \ 1 \leq i \leq n \end{aligned}$$

$f(u) = 3n+1$ and $f(v) = 3n+2$. Since $e_f(1) = 2n+1$ and $e_f(0) = 2n$, f is a difference cordial labeling of G. □

$H_{n,n}$ is a graph with vertex set $\{u_i, v_i : 1 \leq i \leq n\}$ and edge set $\{u_iv_j : 1 \leq i \leq j \leq n\}$.

Theorem 2.23. *$H_{n,n}$ is difference cordial iff $n \leq 6$.*

Proof. Suppose $H_{n,n}$ is difference cordial, then using theorem 2.4, $\frac{n(n+1)}{2} \leq 2(2n) - 1$. It follows that $n \leq 6$. For $n \leq 6$, the difference cordial labeling f is given in table 1. □

n	u_1	u_2	u_3	u_4	u_5	u_6	v_1	v_2	v_3	v_4	v_5	v_6
1	1						2					
2	1	3					2	4				
3	1	3	5				2	4	6			
4	1	3	6	8			2	4	5	7		
5	1	4	6	8	10		2	3	5	7	9	
6	2	4	6	8	10	12	1	3	5	7	9	11

Table 1.

Let G be the graph derived from a wheel W_n by duplicating the hub vertex one or more times. We call G a t-fold wheel if there are t hub vertices, each adjacent to all rim vertices and not adjacent to each other.

Theorem 2.24. *A t-fold wheel G is difference cordial iff $t \leq 2$ and $n = 3$.*

Proof. The 1-fold wheel is a wheel and is difference cordial by theorem 2.7. Let $t > 1$. Clearly G consists of $n + t$ vertices and $nt + n$ edges. Suppose G is difference cordial with $t \geq 3$, then by theorem 2.3, $nt + n \leq 2\,(n + t) - 1$. $\Rightarrow 2n - 1 \geq (n - 2)\,t + n \geq (n - 2)\,3 + n$. It follows that $2n \leq 5$, a contradiction. Suppose G is difference cordial with $t = 2$, then by theorem 2.3, $2n + n \leq 2\,(n + 2) - 1$. This implies $n \leq 3$. The difference cordial labeling of G with $n = 3$ and $t = 2$ is given in figure 7.

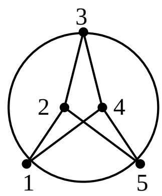

Figure 7

□

Next is the permutation graphs. For any permutation fon $1, 2, \ldots n$, the f-permutation graph on a graph G, $P\,(G, f)$ consists of two disjoint copies of G, say G_1 and G_2, each of which has vertices labeled $v_1, v_2 \ldots v_n$ with n edges obtained by joining each v_i in G_1 to $v_{f(i)}$ in G_2. We shall denote the identity permutation by I.

The product graph $G_1 \times G_2$ is defined as follows: Consider any two vertices $u = (u_1, u_2)$ and $v = (v_1, v_2)$ in $V = V_1 \times V_2$. Then u and v are adjacent in $G_1 \times G_2$ whenever $[u_1 = v_1$ and u_2 adj $v_2]$ or $[u_2 = v_2$ and u_1 adj $v_1]$.

Theorem 2.25. *Let G be (p, q) graph with $q \geq p$. Then for any permutation f, $P\,(G \times K_2, f)$ is not difference cordial.*

Proof. The order and size of $P(G \times K_2, f)$ are $4p$ and $4q+4p$ respectively. Suppose $P(G \times K_2, f)$ is difference cordial with $q \geq p$, then by theorem 2.4, $4q+4p \leq 2(4p)-1$. $\Rightarrow 8p-1 \geq 4q+4p \geq 8p$. $\Rightarrow -1 \geq 0$, a contradiction. □

Theorem 2.26. *For any permutation f, $P\,(W_n, f)$ is not difference cordial.*

Proof. Obviously, the order and size of $P\,(W_n, f)$ are $2n + 2$ and $5n + 1$ respectively. Suppose $P\,(W_n, f)$ is difference cordial. Then by theorem 2.4, $5n + 1 \leq 2\,(2n + 2) - 1$. $\Rightarrow n \leq 2$, a contradiction. □

For a graph G, the splitting graph of G, $S^{'}\,(G)$, is obtained from G by adding for each vertex v of G a new vertex $v^{'}$ so that $v^{'}$ is adjacent to every vertex that is adjacent to v.

Theorem 2.27. *If G is a (p, q) graph and $S^{'}\,(G)$ be the splitting graph of G with $q \geq p$. Then for any permutation f, $P(S^{'}(G), f)$ is not difference cordial.*

Proof. The order and size of $P(S^{'}(G), f)$ are $4p$ and $6q + 2p$ respectively. Suppose $P(S^{'}(G), f)$ is difference cordial, then by theorem 2.4, $6q+2p \leq 2(4p)-1$. $\Rightarrow 6p-1 \geq 6q \geq 6p$. $\Rightarrow -1 \geq 0$, a contradiction. □

The helm H_n is the graph obtained from a wheel by attaching a pendant edge at each vertex of the n-cycle. A flower Fl_n is the graph obtained from a helm by joining each pendant vertex to the central vertex of the helm.

Theorem 2.28. *For any permutation f, $P(Fl_n, f)$ is not difference cordial.*

Proof. The order and size of $P(Fl_n, f)$ are $4n+2$ and $10n+1$ respectively. Suppose $P(Fl_n, f)$ is difference cordial. Then by theorem 2.4, $10n+1 \leq 2(4n+2)-1$. $\Rightarrow n \leq 1$, a contradiction. □

Theorem 2.29. $P(P_{2k}, f)$ *is difference cordial where* $f = (1\,2)(3\,4)\ldots(k\;k{+}1)\ldots(2k-1\;2k)$.

Proof. Let u_i and v_i be the vertices in the first and second copies of P_{2k}. Define, $f : V(P(P_{2k}, f)) \to \{1, 2, \ldots, 4k\}$ by $f(u_i) = i,\ 1 \le i \le 2k$,

$$\begin{array}{rcllll} f(v_{4i-3}) & = & 2k + 4i - 3 & 1 \le i \le \frac{k+1}{2} & \text{if} & 2k \equiv 2 \pmod 4 \\ & & & 1 \le i \le \frac{k}{2} & \text{if} & 2k \equiv 0 \pmod 4. \\ f(v_{4i-2}) & = & 2k + 4i - 2 & 1 \le i \le \frac{k+1}{2} & \text{if} & 2k \equiv 2 \pmod 4 \\ & & & 1 \le i \le \frac{k}{2} & \text{if} & 2k \equiv 0 \pmod 4. \\ f(v_{4i-1}) & = & 2k + 4i & 1 \le i \le \frac{k-1}{2} & \text{if} & 2k \equiv 2 \pmod 4 \\ & & & 1 \le i \le \frac{k}{2} & \text{if} & 2k \equiv 0 \pmod 4. \\ f(v_{4i}) & = & 2k + 4i - 1 & 1 \le i \le \frac{k-1}{2} & \text{if} & 2k \equiv 2 \pmod 4 \\ & & & 1 \le i \le \frac{k}{2} & \text{if} & 2k \equiv 0 \pmod 4. \end{array}$$

Since, $e_f(0) = e_f(1) = 3k - 1$, f is a difference cordial labeling. □

Theorem 2.30. *$P(P_n, I)$ is difference cordial.*

Proof. Since $P(P_n, I) \cong L_n$, proof follows from theorem 2.9. □

Theorem 2.31. *$P(C_n, I)$ is difference cordial.*

Proof. Since $P(C_n, I) \cong C_n \times P_2$, proof follows from theorem 2.10. □

Theorem 2.32. *$P(K_n, I)$ is difference cordial iff $n \le 3$.*

Proof. Since $P(K_1, I) \cong K_2$, $P(K_2, I) \cong C_4$ and $P(K_n, I) \cong C_3 \times P_2$, $P(K_n, I), n \le 3$ is difference cordial. The order and size of $P(K_n, I)$ are $2n$ and n^2 respectively. Suppose $P(K_n, I)$ is difference cordial, then by theorem 2.4, $n^2 \le 4n - 1$. It follows that $n \le 3$. □

The corona of G with H, $G \odot H$ is the graph obtained by taking one copy of G and p copies of H and joining the i^{th} vertex of G with an edge to every vertex in the i^{th} copy of H. $P_n \odot K_1$ is called the comb and $P_n \odot 2K_1$ is called the double comb.

Theorem 2.33. *$P(P_n \odot K_1, I)$ is difference cordial.*

Proof. Let $V(P(P_n \odot K_1, I)) = \{u_i, v_i, w_i, x_i : 1 \le i \le n\}$ and $E(P(P_n \odot K_1, I)) = \{u_iv_i, v_iw_i, w_ix_i, u_ix_i : 1 \le i \le n\} \cup \{u_iu_{i+1}, x_ix_{i+1} : 1 \le i \le n - 1\}$. Define an injective map $f : V(P(P_n \odot K_1, I)) \to \{1, 2 \ldots 4n\}$ by

$$\begin{array}{rcll} f(u_i) & = & 4i - 3 & 1 \le i \le n \\ f(v_i) & = & 4i - 2 & 1 \le i \le n \\ f(w_i) & = & 4i - 1 & 1 \le i \le n - 1 \\ f(x_i) & = & 4i & 1 \le i \le n - 1 \end{array}$$

$f(w_n) = 4n$ and $f(x_n) = 4n - 1$. Since $e_f(0) = e_f(1) = 3n - 1$, f is a difference cordial labeling of $P(P_n \odot K_1, I)$. □

Theorem 2.34. *$P(P_n \odot 2K_1, I)$ is difference cordial.*

Proof. Let $V(P(P_n \odot 2K_1, I)) = \{u_i, v_i, w_i, x_i, y_i, z_i : 1 \le i \le n\}$ and $E(P(P_n \odot 2K_1, I)) = \{u_iw_i, u_ix_i, w_ix_i, w_iy_i, y_iv_i, u_iv_i, x_iz_i, z_iv_i : 1 \le i \le n\} \cup \{u_iu_{i+1}, v_iv_{i+1} : 1 \le i \le n - 1\}$. Define a one to one map $f : V(P(P_n \odot 2K_1, I)) \to \{1, 2 \ldots 6n\}$ as follows:

$$\begin{array}{rcll} f(u_i) & = & 6i - 5 & 1 \le i \le \left\lceil \frac{n}{2} \right\rceil \\ f(w_i) & = & 6i - 4 & 1 \le i \le \left\lceil \frac{n}{2} \right\rceil \\ f(x_i) & = & 6i & 1 \le i \le \left\lfloor \frac{n-1}{2} \right\rfloor \\ f(y_i) & = & 6i - 3 & 1 \le i \le \left\lceil \frac{n}{2} \right\rceil \\ f(z_i) & = & 6i - 1 & 1 \le i \le \left\lceil \frac{n}{2} \right\rceil \\ f(v_i) & = & 6i - 2 & 1 \le i \le \left\lceil \frac{n}{2} \right\rceil \\ f\left(u_{\left\lceil \frac{n}{2} \right\rceil + i}\right) & = & 6\left\lceil \frac{n}{2} \right\rceil + 5i - 5 & 1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor \\ f\left(w_{\left\lceil \frac{n}{2} \right\rceil + i}\right) & = & 6\left\lceil \frac{n}{2} \right\rceil + 5i - 4 & 1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor \\ f\left(y_{\left\lceil \frac{n}{2} \right\rceil + i}\right) & = & 6\left\lceil \frac{n}{2} \right\rceil + 5i - 3 & 1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor \\ f\left(v_{\left\lceil \frac{n}{2} \right\rceil + i}\right) & = & 6\left\lceil \frac{n}{2} \right\rceil + 5i - 2 & 1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor \\ f\left(z_{\left\lceil \frac{n}{2} \right\rceil + i}\right) & = & 6\left\lceil \frac{n}{2} \right\rceil + 5i - 1 & 1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor \\ f\left(x_{\left\lceil \frac{n}{2} \right\rceil + i}\right) & = & 6\left\lceil \frac{n}{2} \right\rceil + 5\left\lfloor \frac{n}{2} \right\rfloor + i - 1 & 1 \le i \le \left\lceil \frac{n+1}{2} \right\rceil. \end{array}$$

The following table 2 shows that f is a difference cordial labeling. ◻

Nature of n	$e_f(0)$	$e_f(1)$
$n \equiv 0 \pmod 2$	$\frac{9n-2}{2}$	$\frac{9n-2}{2}$
$n \equiv 1 \pmod 2$	$\frac{9n-3}{2}$	$\frac{9n-1}{2}$

Table 2.

Theorem 2.35. *$P(K_2 + mK_1, I)$ is difference cordial iff $m \leq 3$.*

Proof. The order and size of $P(K_2 + mK_1, I)$ are $2m + 4$ and $5m + 4$ respectively. By theorem 2.4, $5m + 4 \leq 2(2m + 4) - 1$. It follows that $m \leq 3$. The difference cordial of $K_2 + K_1$, $K_2 + 2K_1$ and $K_2 + 3K_1$ are shown in figure 8.

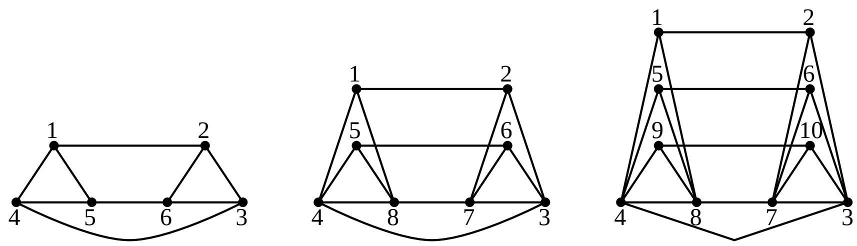

Figure 8

◻

Theorem 2.36. *Let C_n be the cycle $u_1u_2 \ldots u_nu_1$. Let G^* be the graph with $V(G^*) = V(C_n) \cup \{v_i, w_i : 1 \leq i \leq n\}$ and $E(G^*) = E(C_n) \cup \{u_iv_i, u_{i+1(\text{mod } n)}w_i, v_iw_i : 1 \leq i \leq n\}$. Then G^* is difference cordial.*

Proof. Define a function $f : V(G^*) \to \{1, 2 \ldots 3n\}$ as follows:

$$\begin{aligned} f(u_i) &= i & 1 \leq i \leq n \\ f(v_i) &= n + 2i + 1 & 1 \leq i \leq n-1 \\ f(w_i) &= n + 2i + 2 & 1 \leq i \leq n-1 \end{aligned}$$

$f(v_n) = n + 1$ and $f(w_n) = n + 2$. Since $e_f(0) = e_f(1) = 2n$, f is a difference cordial labeling of G^*. ◻

Theorem 2.37. *$P(K_{m,n}, I)$ $(m, n > 1)$ is difference cordial iff $m = n = 2$ and $n = 3, 4, 5$.*

Proof. The order and size of $P(K_{m,n}, I)$ are $2m + 2n$ and $2mn + m + n$ respectively. Suppose $P(K_{m,n}, I)$ is difference cordial, then by theorem 2.4, $2mn + m + n \leq 2(2m + 2n) - 1$. $\Rightarrow 2mn \leq 3m + 3n - 1 \longrightarrow$ (1).
Case 1. $m = n$.
(1) $\Rightarrow 2m^2 \leq 6m - 1$. $\Rightarrow m = n = 2$.
Case 2. $m \neq n$.
Assume $m > n \geq 2$. (1) $\Rightarrow 0 \leq -2mn + 3m + 3n - 1 < -2mn + 6m - 1$. $\Rightarrow 6m - 1 > 2mn \longrightarrow$ (2).
Subcase 1. $n \geq 3$.
(2) $\Rightarrow 6m - 1 \geq 6m$. $\Rightarrow -1 \geq 0$, a contradiction.
Subcase 2. $n = 2$.
Here $p = 2m + 4$ and $q = 5m + 2$. Suppose f is difference cordial, then by theorem 2.4, $5m + 2 \leq 2(2m + 4) - 1$. This implies $m \leq 5$. Since $P(K_{2,2}, I) \cong C_4 \times P_2$, by theorem 2.10, $P(K_{2,2}, I)$ is difference cordial. The difference cordial labeling of $P(K_{2,3}, I)$, $P(K_{2,4}, I)$ and $P(K_{2,5}, I)$ is shown in figure 9.

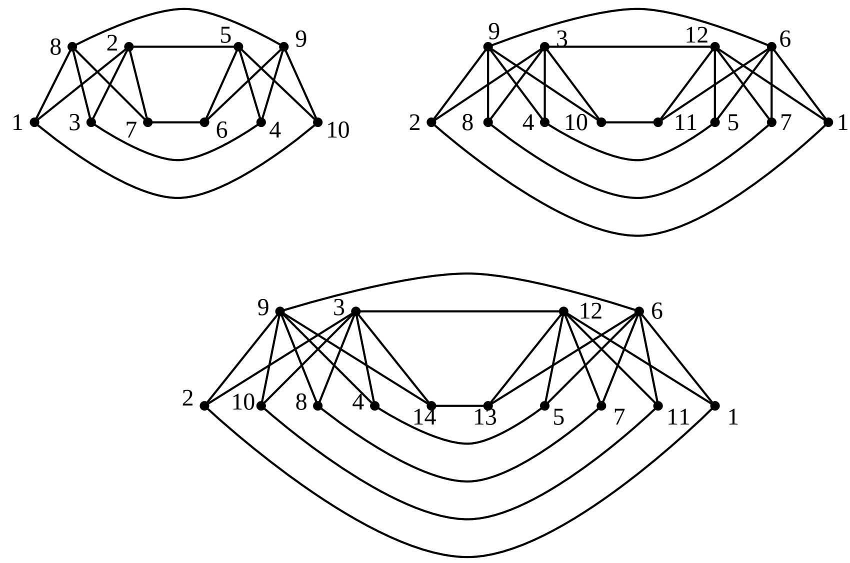

Figure 9

$\square$

Finally we investigate the difference cordial labeling behavior of special graphs which are generated from cycle.

Theorem 2.38. *Let C_n be the cycle $u_1u_2 \dots u_nu_1$. Let G be a graph with $V(G) = V(C_n) \cup \{w_i : 1 \le i \le n\}$ and $E(G) = E(C_n) \cup \{u_iw_i, u_{i+1(\text{mod } n)}w_i : 1 \le i \le n\}$. Then G is difference cordial.*

Proof. Define a map $f : V(G) \to \{1, 2 \dots 2n\}$ as follows:
Case 1. n is even.

$$\begin{array}{llll} f(u_i) & = & 2i-1 & 1 \le i \le \frac{n+4}{2} \\ f(u_{\frac{n+4}{2}+i}) & = & \frac{3n+4}{2}+i & 1 \le i \le \frac{n-4}{2} \\ f(w_i) & = & 2i & 1 \le i \le \frac{n+2}{2} \\ f(w_{n-i}) & = & n+2+i & 1 \le i \le \frac{n-2}{2}. \end{array}$$

Case 2. n is odd.

$$\begin{array}{llll} f(u_i) & = & 2i-1 & 1 \le i \le \frac{n+5}{2} \\ f(u_{\frac{n+5}{2}+i}) & = & \frac{3n+5}{2}+i & 1 \le i \le \frac{n-5}{2} \\ f(w_i) & = & 2i & 1 \le i \le \frac{n+3}{2} \\ f(w_{n-i}) & = & n+4+i & 1 \le i \le \frac{n-3}{2}. \end{array}$$

The table 3 gives the nature of the edge condition of the above labeling f. It follows that f is a difference cordial labeling. $\square$

Values of n	$e_f(0)$	$e_f(1)$
$n \equiv 0 \,(\text{mod } 2)$	$\frac{3n}{2}$	$\frac{3n}{2}$
$n \equiv 1 \,(\text{mod } 2)$	$\frac{3n-1}{2}$	$\frac{3n+1}{2}$

Table 3.

References

[1] I. Cahit, Cordial graphs: a weaker version of graceful and harmonious graphs, *Ars combin.,* **23** (1987) 201-207.

[2] J. A. Gallian, A Dynamic survey of graph labeling, *The Electronic Journal of Combinatorics,* **18** (2012) #Ds6.

[3] F. Harary, Graph theory, *Addision wesley, New Delhi* (1969).

[4] Y. S. Ho, S. M. Lee, and S. C. Shee, Cordial labellings of unicyclic graphs and generalized Petersen graphs, *Congr. Numer.,* **68** (1989) 109-122.

[5] S. M. Lee and A. Liu, A construction of cordial graphs from smaller cordial graphs, *Ars Combin.,* **32** (1991) 209-214.

[6] R. Ponraj, S. Sathish Narayanan and R. Kala, Difference cordial labeling of graphs, *Global Journal of Mathematical Sicences: Theory and Practical,* **3**(2013), 192-201.

[7] R. Ponraj and S. Sathish Narayanan, Further Results on Difference cordial labeling of corona graphs, *The Journal of The Indian Academy of Mathematics,* **35(@)**(2013), 217-235.

[8] R. Ponraj, S. Sathish Narayanan and R. Kala, Difference cordial labeling of graphs obtained from double snakes, *International Journal of Mathematics Research,* **5**(2013), 317-322.

[9] R. Ponraj and S. Sathish Narayanan, Difference cordiality of some graphs obtained from double alternate snake graphs, *Global Journal of Mathematical Sciences: Theory and Practical,* **5**(2013), 167-175.

[10] R. Ponraj, S. Sathish Narayanan and R. Kala Difference cordiality of product related graphs, (communicated).

[11] A. Rosa, On certain valuations of the vertices of a graph, *Theory of Graphs (Internat. Symposium, Rome, July 1966),* Gordon and Breach, N. Y. and Dunod Paris (1967) 349-355.

[12] M. A. Seoud and A. E. I. Abdel Maqsoud, On cordial and balanced labelings of graphs, *J. Egyptian Math. Soc.,* **7**(1999) 127-135.

[13] M. A. Seoud and A. E. I. Abdel Maqsoud, On 3-equitable and magic labelings, preprint.

[14] M. A. Seoud, A. T. Diab, and E. A. Elsahawi, On strongly-C harmonious, relatively prime, odd graceful and cordial graphs, *Proc. Math. Phys. Soc. Egypt,* No. **73** (1998) 33-55.

[15] S. K. Vaidya, G. Ghodasara, S. Srivastav, and V. Kaneria, Cordial labeling for two cycle related graphs, *Math. Student,* **76** (2007) 237-246.

[16] S. K. Vaidya, G. Ghodasara, S. Srivastav, and V. Kaneria, Some new cordial graphs, *Internat. J. Scientific Computing,* **2** (2008) 81-92.

[17] S. K. Vaidya, K. Kanani, S. Srivastav, and G. Ghodasara, Baracentric subdivision and cordial labeling of some cycle related graphs, *Proceedings of the First International Conference on Emerging Technologies and Applications in Engineering, Technology and Sciences,* (2008) 1081-1083.

[18] S. K. Vaidya, K. Kanani, P. Vihol, and N. Dani, Some cordial graphs in the context of barycentric subdivision, *Int. J. Comtemp. Math. Sciences,* **4** (2009) 1479-1492.

[19] S. K. Vaidya and P. L. Vihol, Cordial labeling for middle graph of some graphs, *Elixir Discrete Math.,* **34C** (2011) 2468-2475.

[20] M. Z. Youssef, Graph operations and cordiality, *Ars Combin.,* **97** (2010) 161-174.

Author information

R. Ponraj, S. Sathish Narayanan, Department of Mathematics, Sri Paramakalyani College, Alwarkurichi-627 412, Tamil Nadu, India.
E-mail: `ponrajmaths@gmail.com; sathishrvss@gmail.com`

R. Kala, Department of Mathematics, Manonmaniam Sundaranar University, Tirunelveli-627 012, Tamil Nadu, India.
E-mail: `karthipyi91@yahoo.co.in`

Received: July 26, 2013.

Accepted: January 23, 2014.